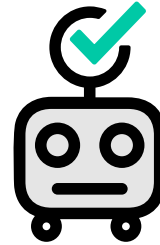


# Checkmate



## Checkmate: Breaking the Memory Wall with Optimal Tensor Rematerialization

Paras Jain

*Joint work with:* Ajay Jain, Ani Nrusimha, Amir Gholami,  
Pieter Abbeel, Kurt Keutzer, Ion Stoica, Joseph Gonzalez



## BigGAN (2018)

Image generation



Brock et al. 2019

## VideoBERT (2019)

Video generation



Sun et al. 2019

## GPT-2 (2019)

Text generation

SYSTEM PROMPT (HUMAN-WRITTEN)

*In a shocking finding, scientist discovered a herd of unicorns living in a remote, previously unexplored valley, in the Andes Mountains. Even more surprising to the researchers was the fact that the unicorns spoke perfect English.*

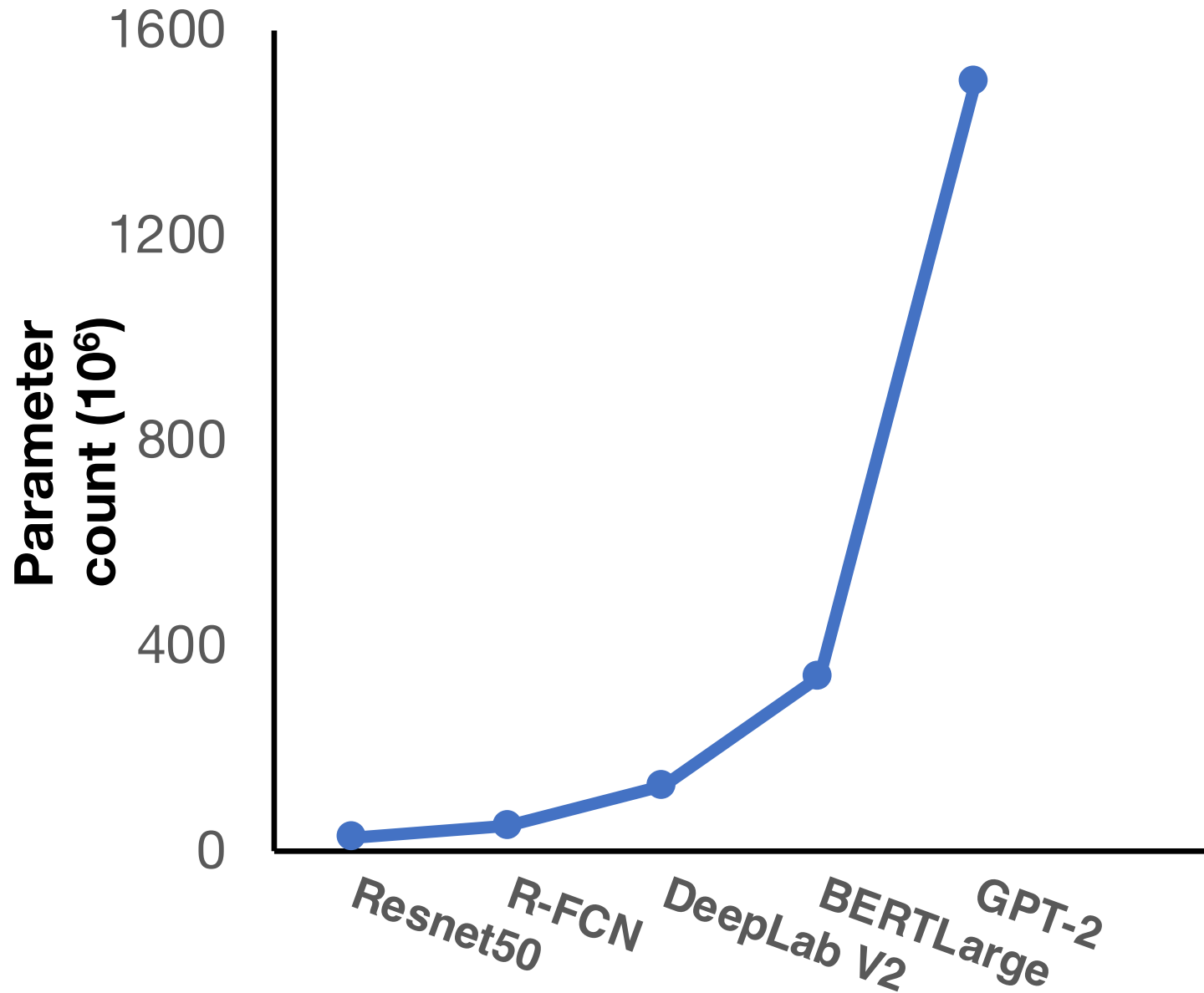
MODEL COMPLETION (MACHINE-WRITTEN, 10 TRIES)

The scientist named the population, after their distinctive horn, Ovid's Unicorn. These four-horned, silver-white unicorns were previously unknown to science.

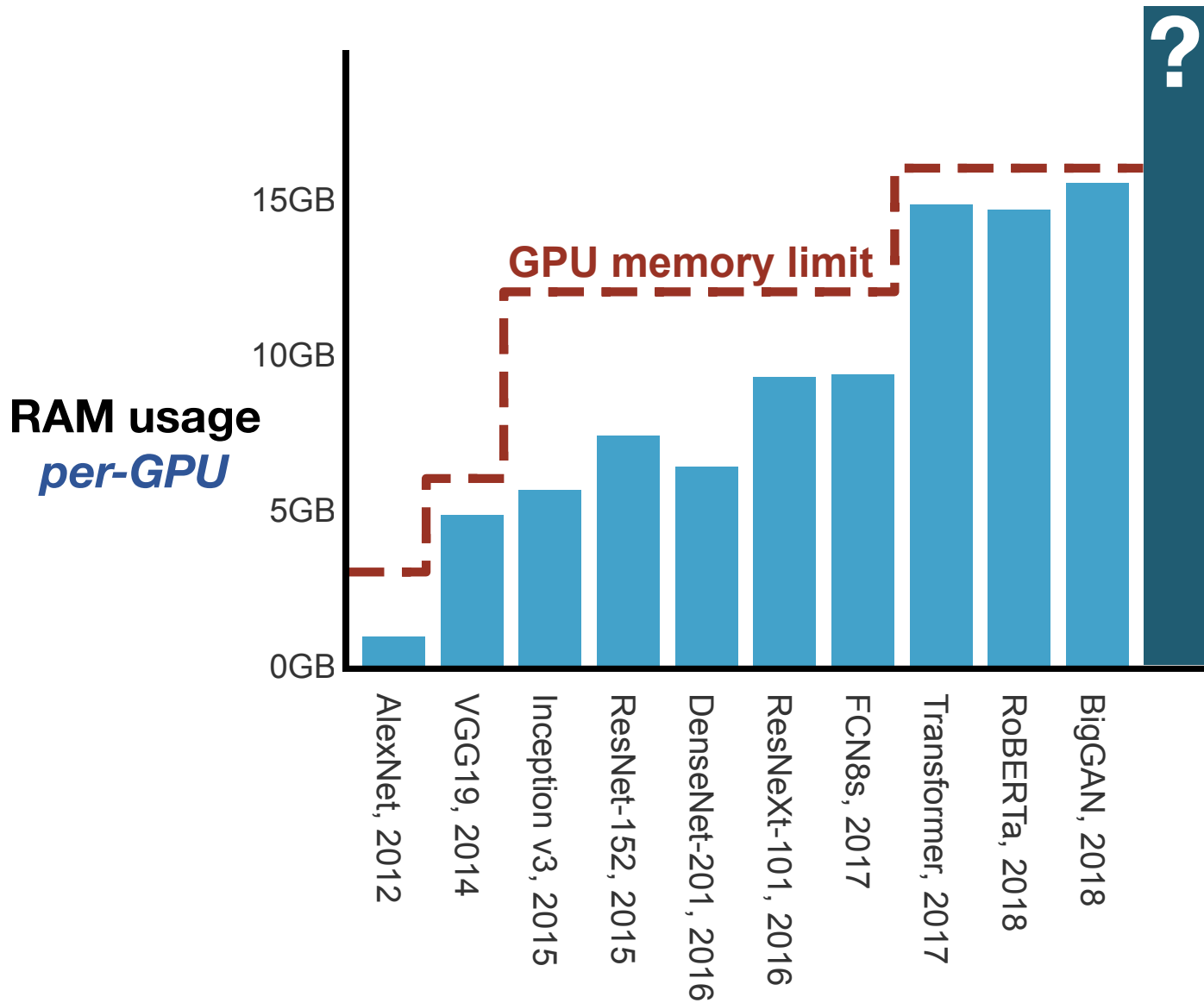
Now, after almost two centuries, the mystery of what sparked this odd phenomenon is finally solved.

Dr. Jorge Pérez, an evolutionary biologist from the University of La Paz, and several

Radford et al. 2019



**Emerging trend:**  
Rapid **growth in model size**



State-of-the-art models have hit a **memory capacity wall**.

**Cited memory as limiting factor**

Chen et al. 2016  
 Gomez et al. 2017  
 Pohlen et al. 2017

Liu et al. 2019  
 Dai et al. 2019  
 Child et al. 2019

**Limited GPU memory is slowing progress in new deep learning models!**



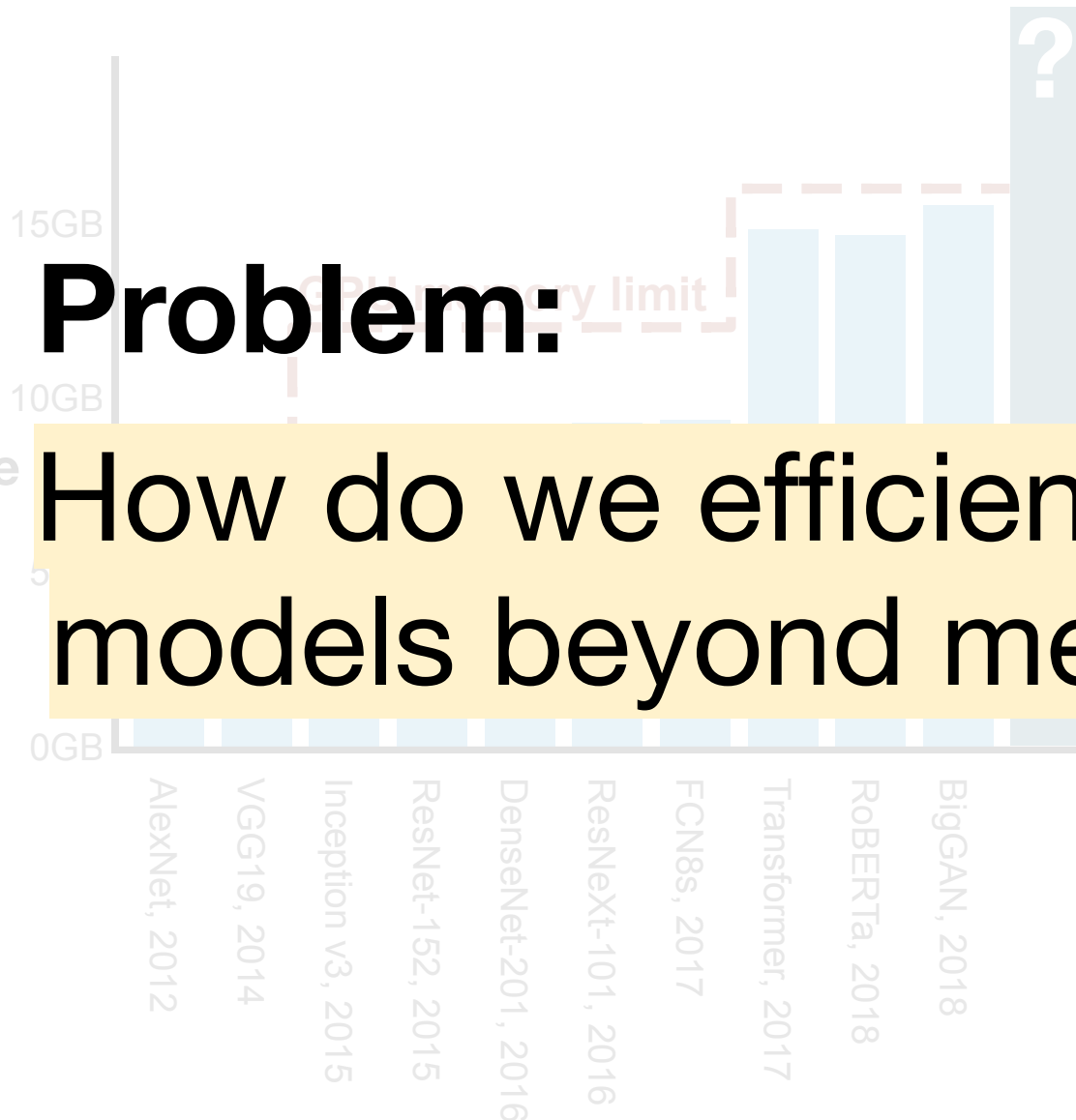
# Problem:

How do we efficiently train large models beyond memory limits?

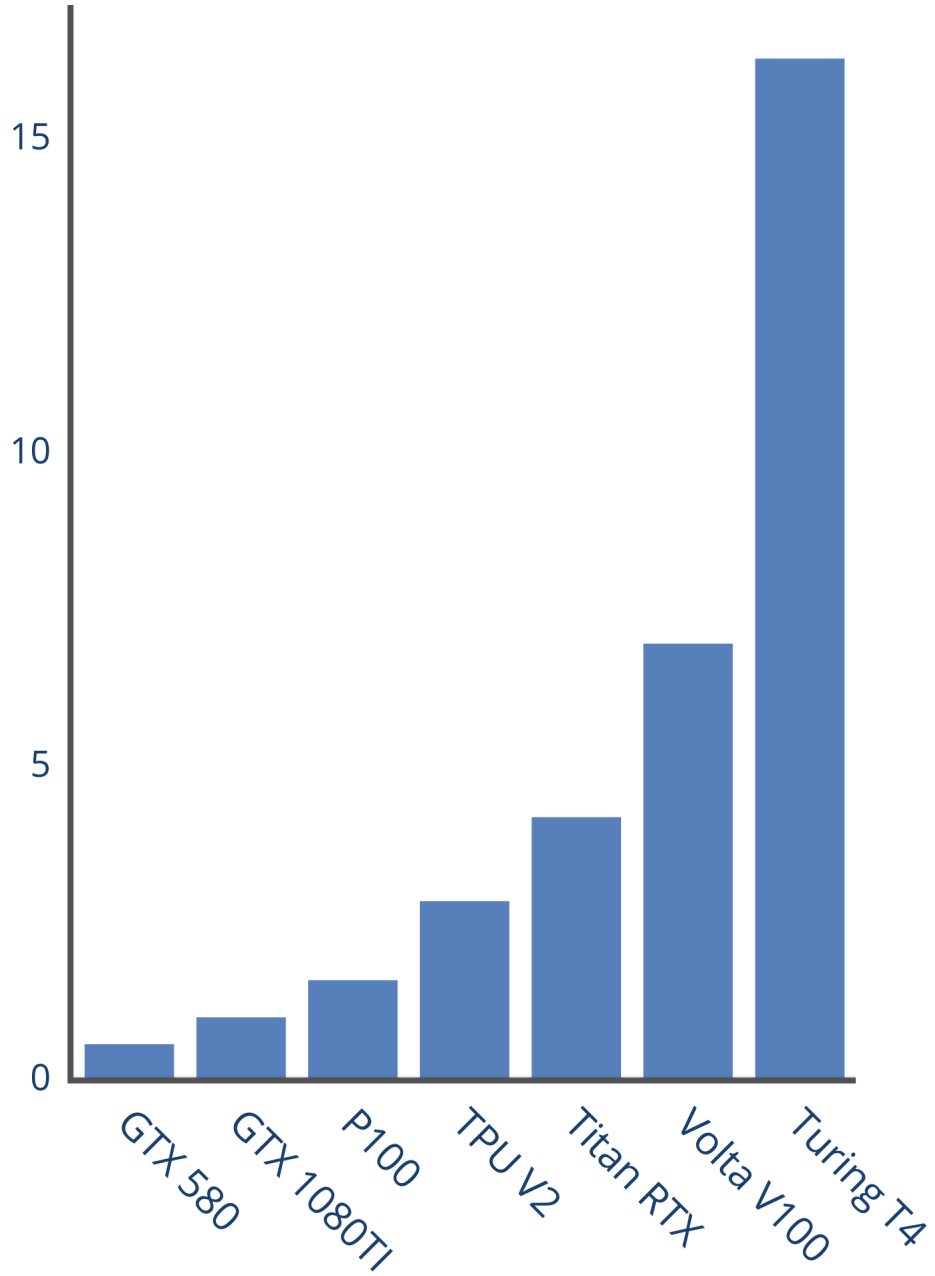
State-of-the-art models have hit a **memory capacity wall**.

Cited memory as limiting factor

Limited GPU memory is slowing progress in new deep learning models!



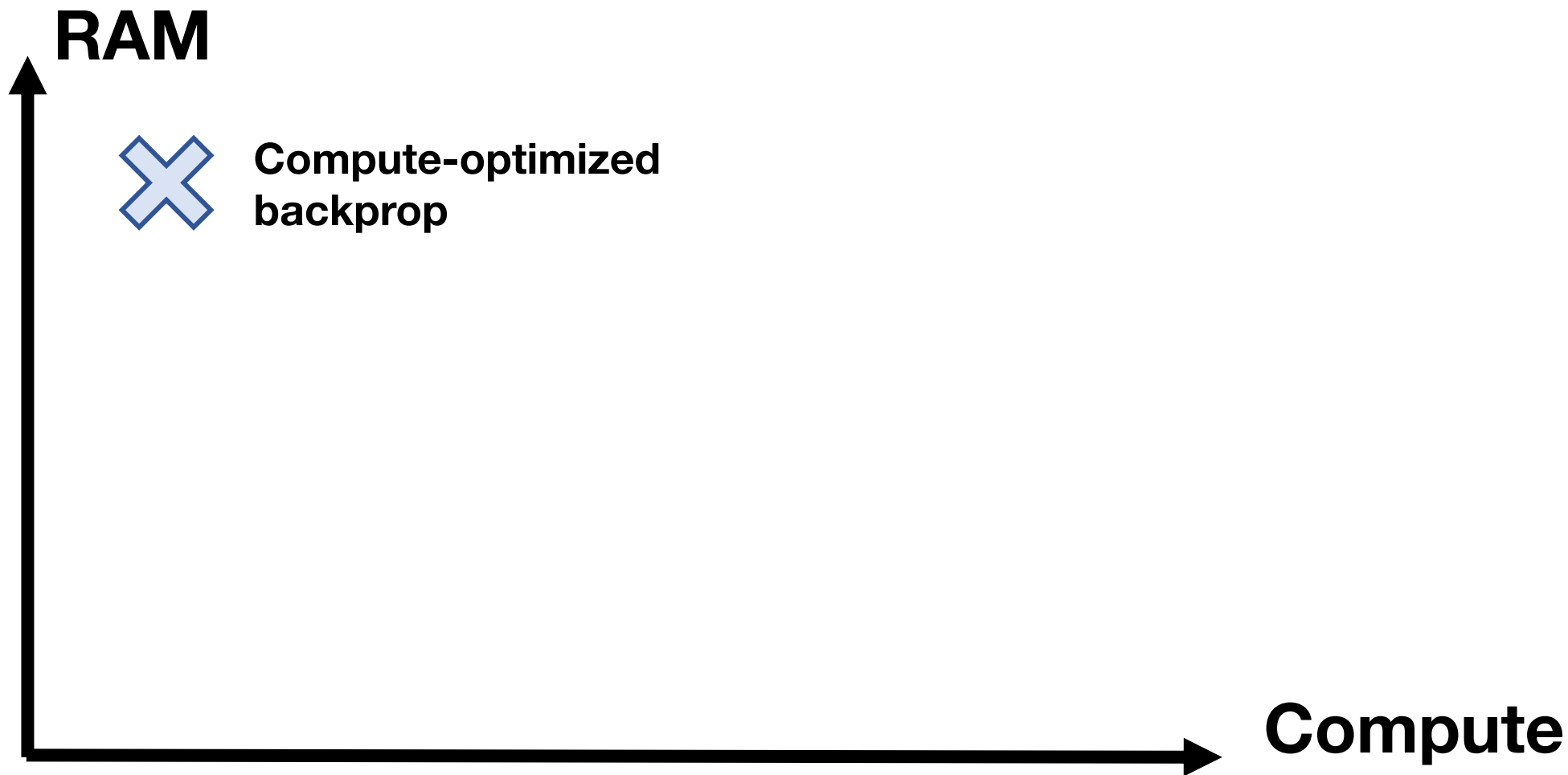
**TOPS  
per GiB  
capacity**



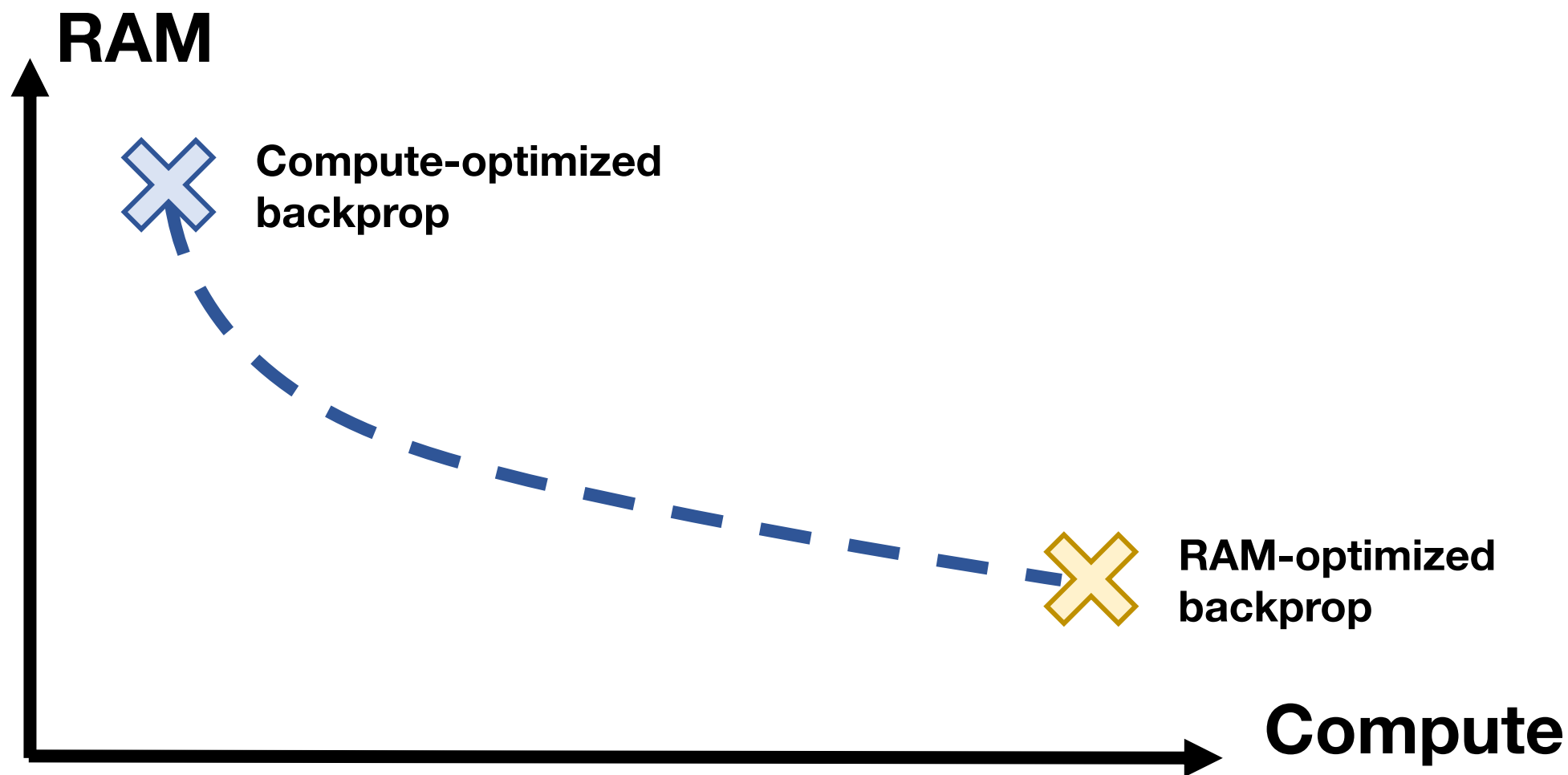
**Compute is outstripping DRAM  
capacity growth**



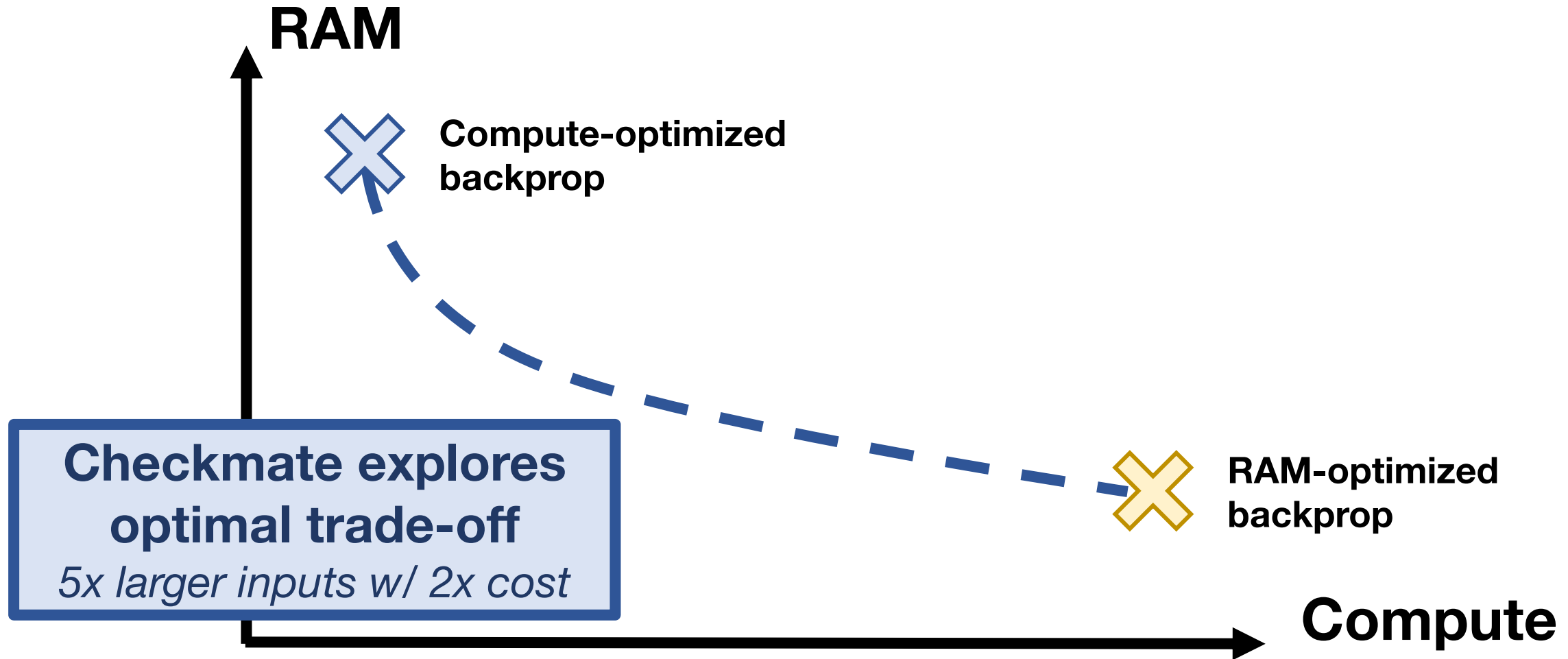
# Backprop is optimized for **compute efficiency**, not **RAM usage**



**Ideal:** scalable algorithm for backprop that adapts to RAM constraints

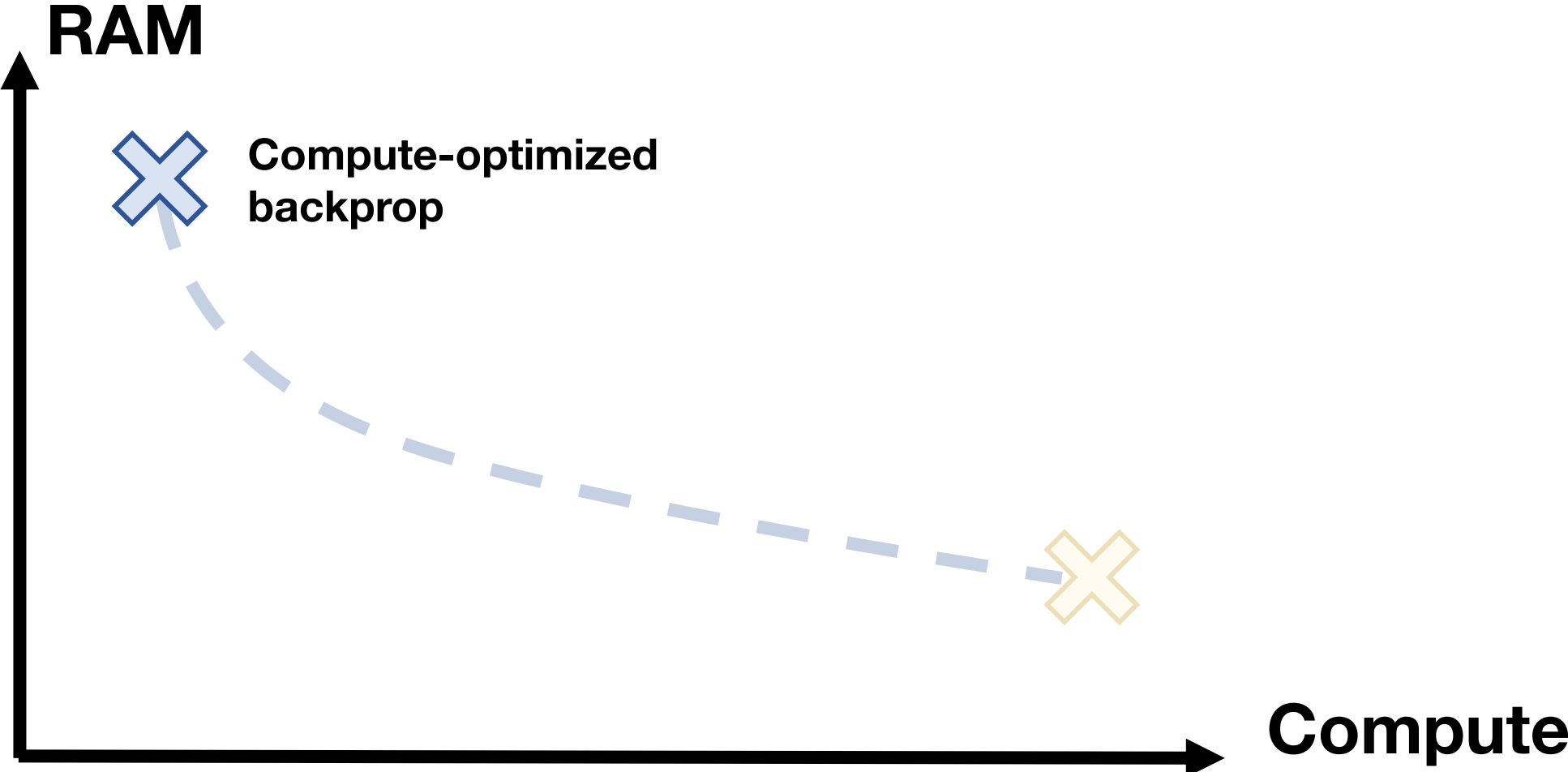


# This work: optimal space-time tradeoff for backpropagation



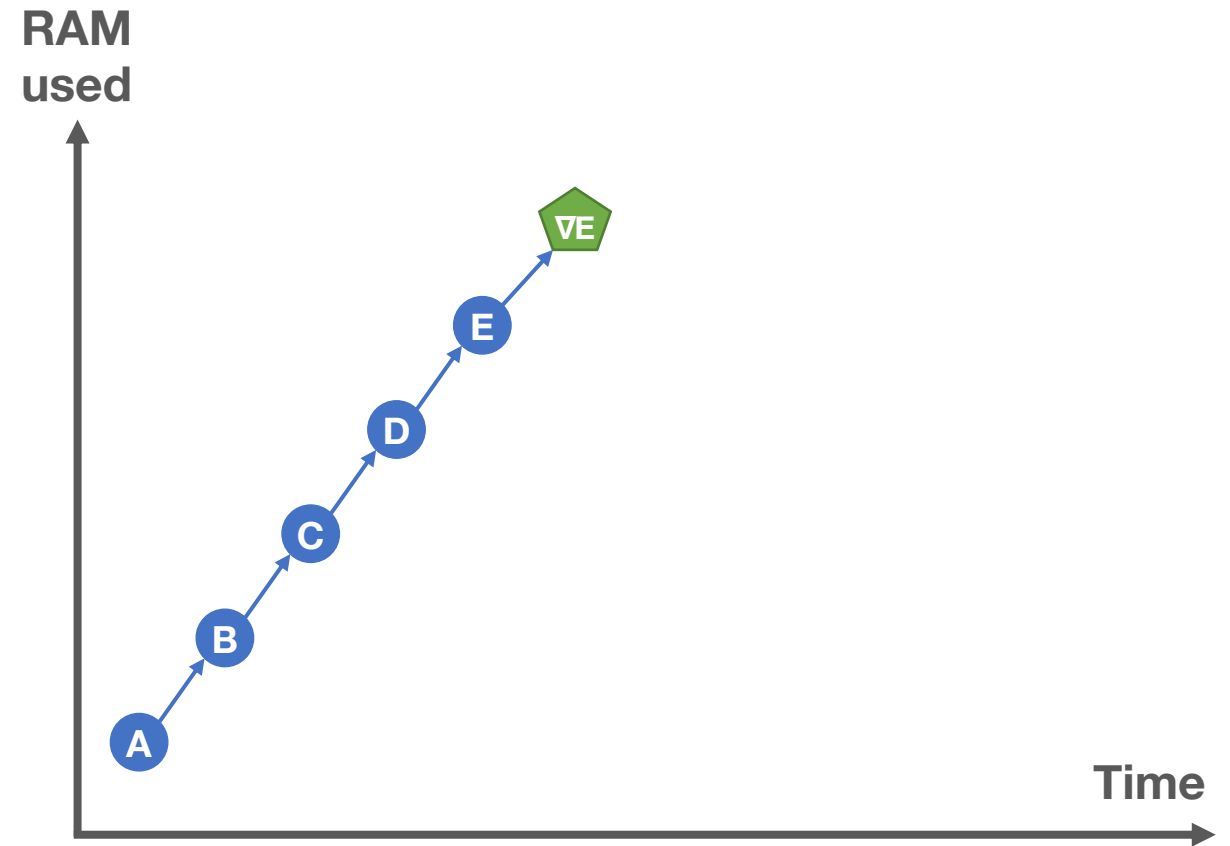
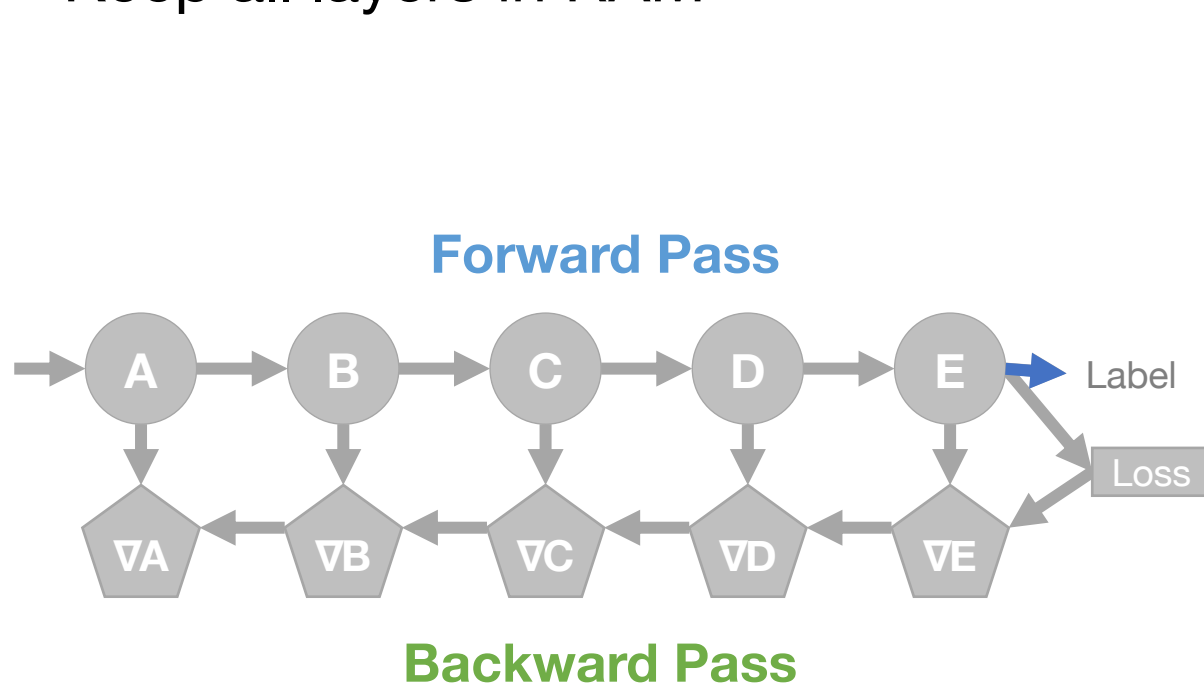
# RAM-hungry backprop policy

Keep all layers in RAM



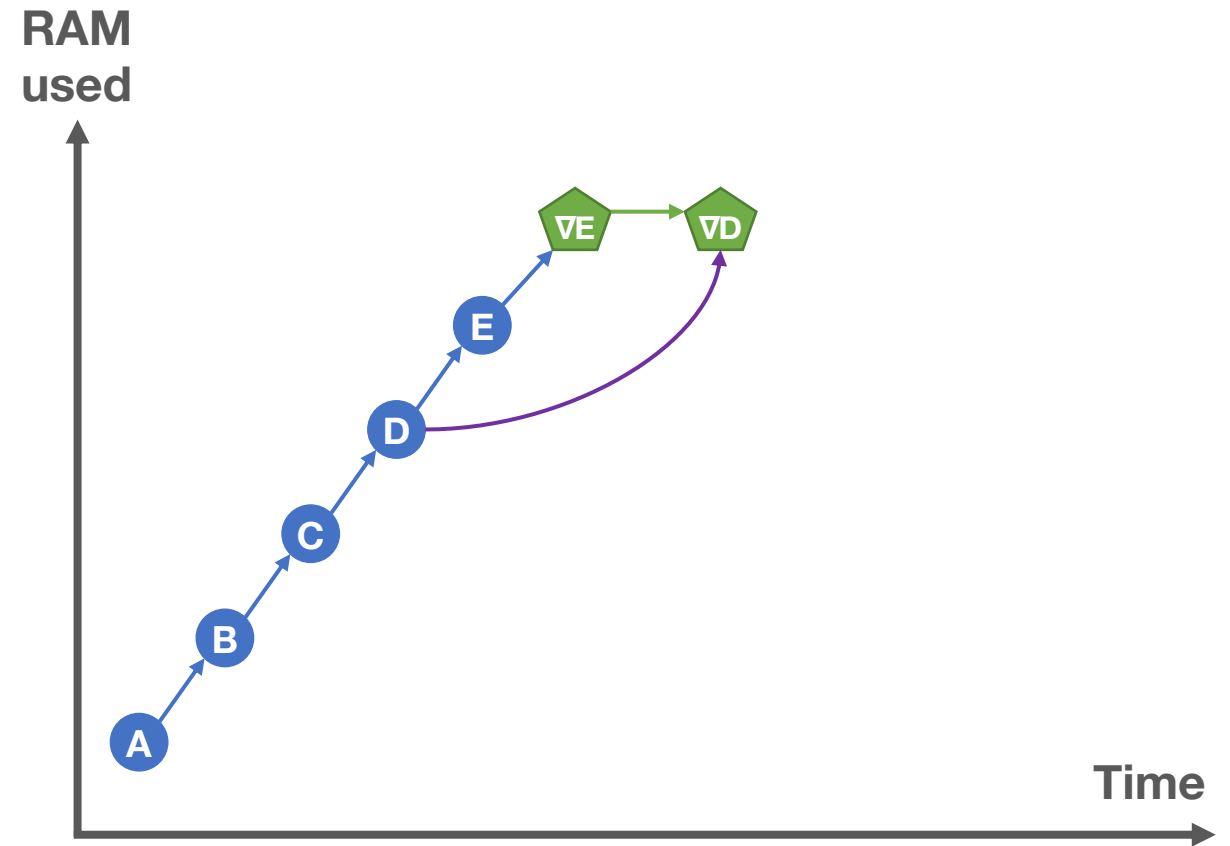
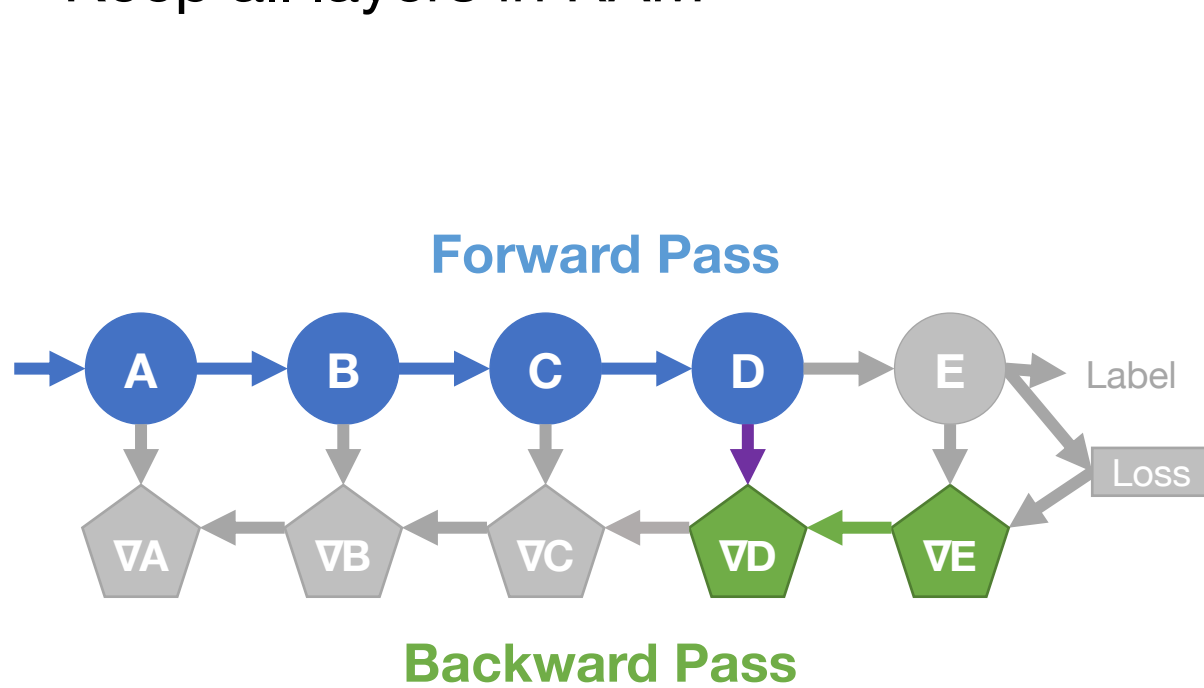
# RAM-hungry backpropagation policy

Keep all layers in RAM



# RAM-hungry backpropagation policy

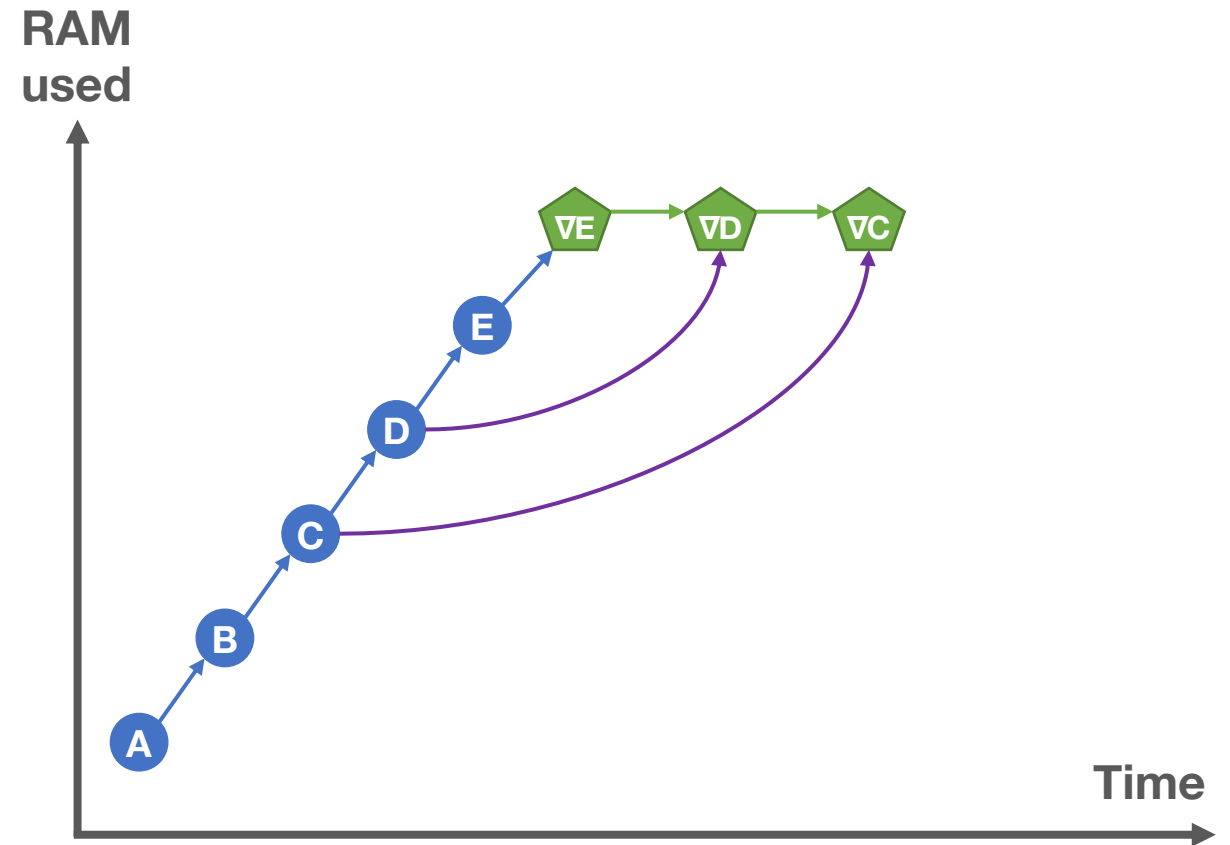
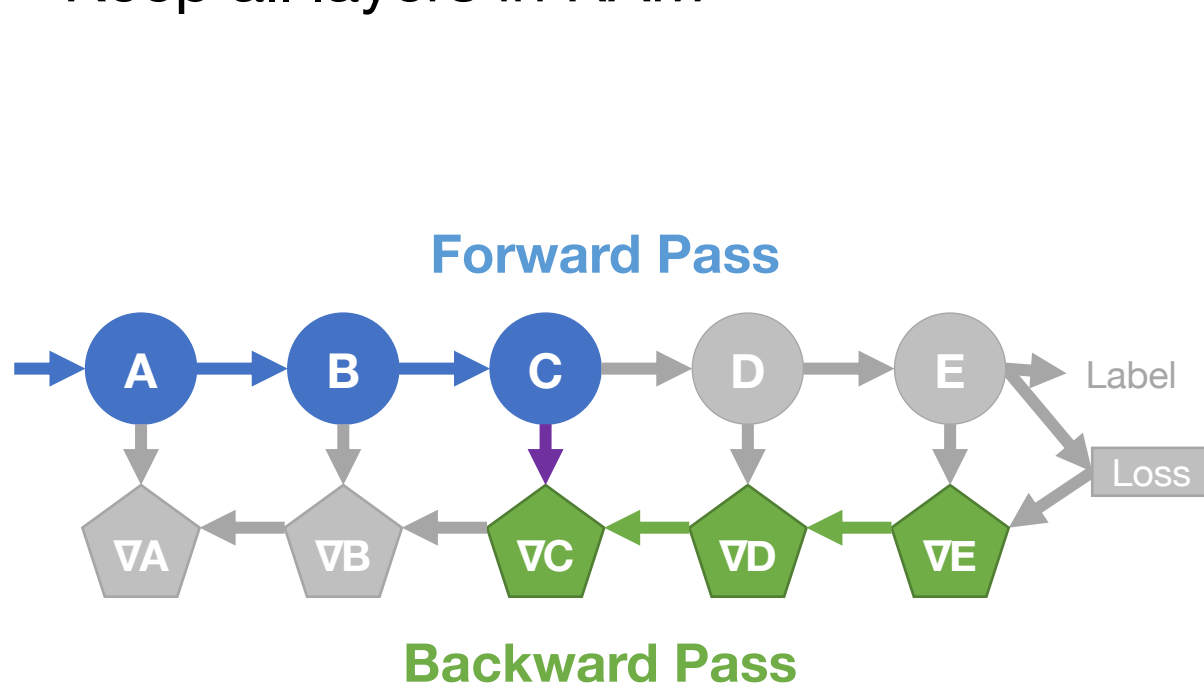
Keep all layers in RAM





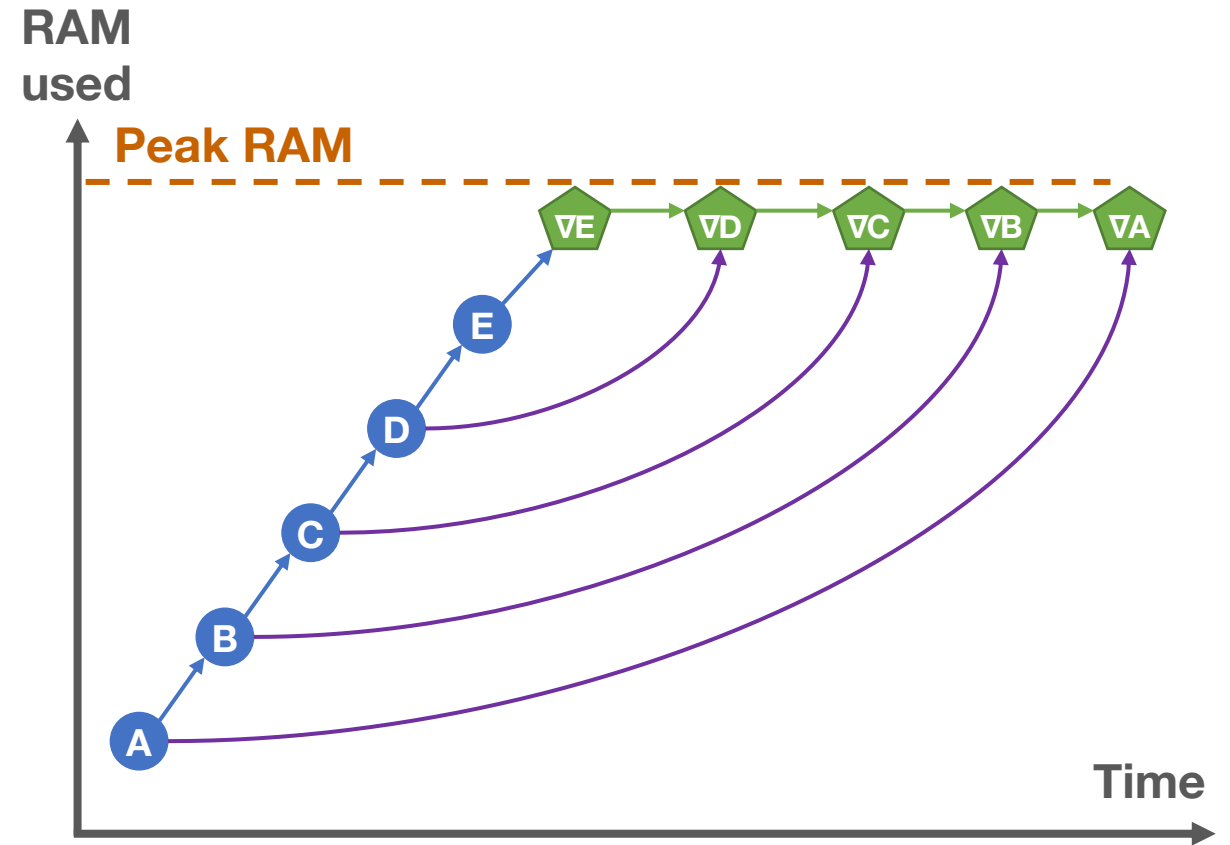
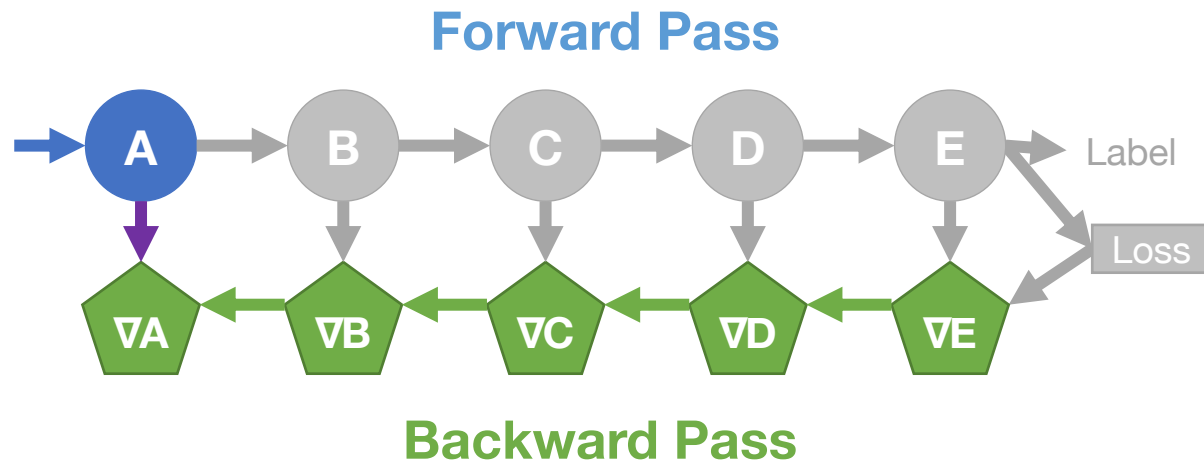
# RAM-hungry backpropagation policy

Keep all layers in RAM



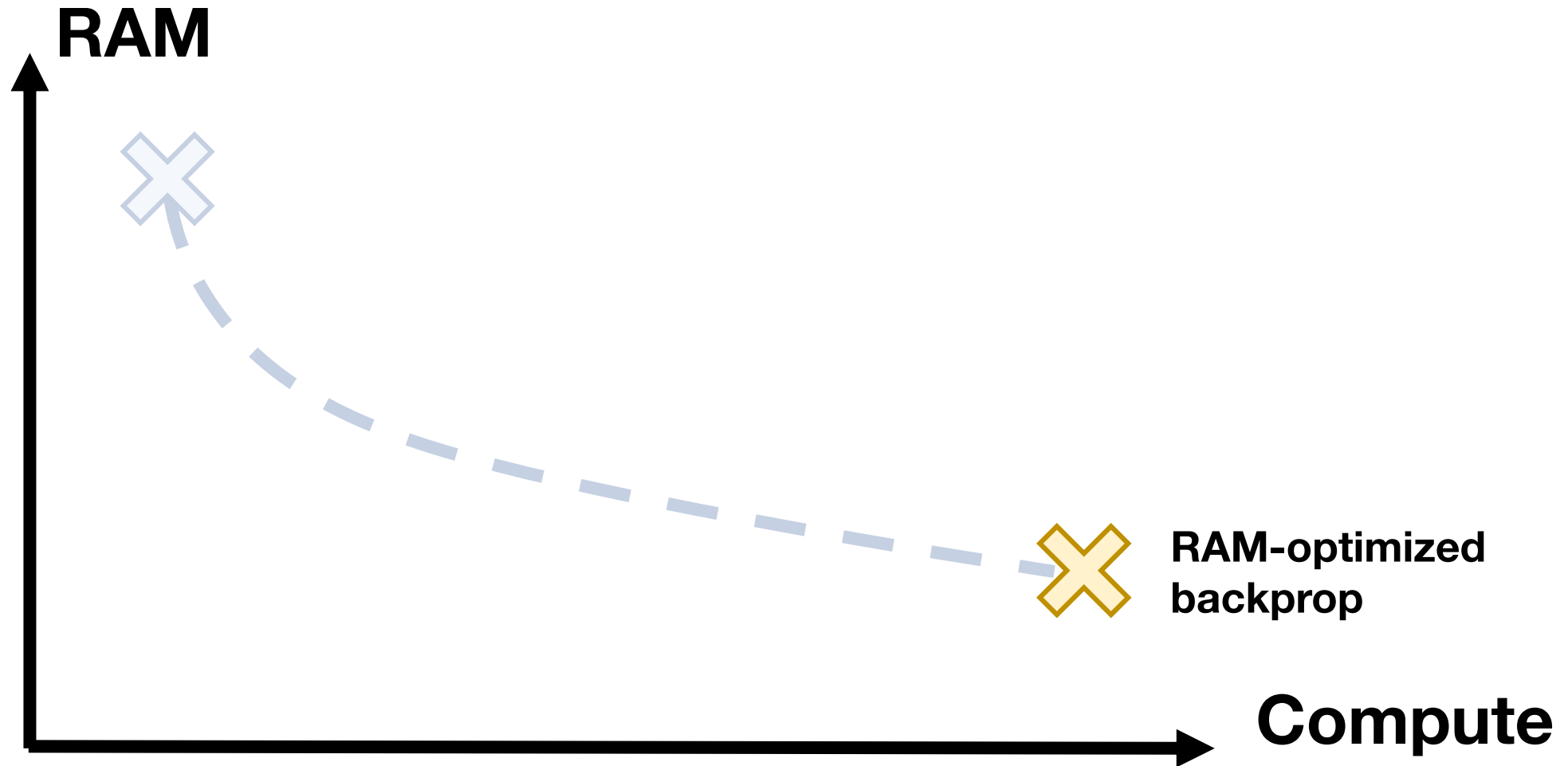
# RAM-hungry backpropagation policy

Keep all layers in RAM



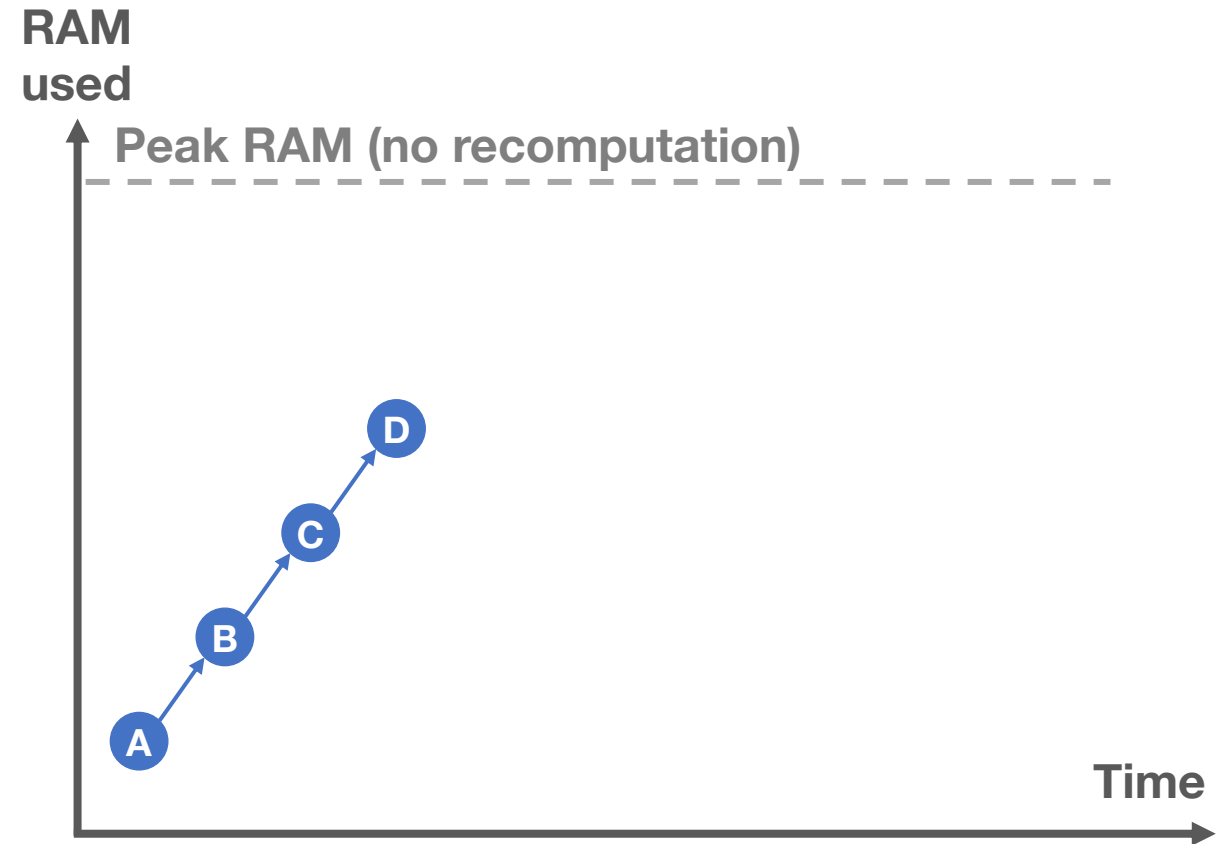
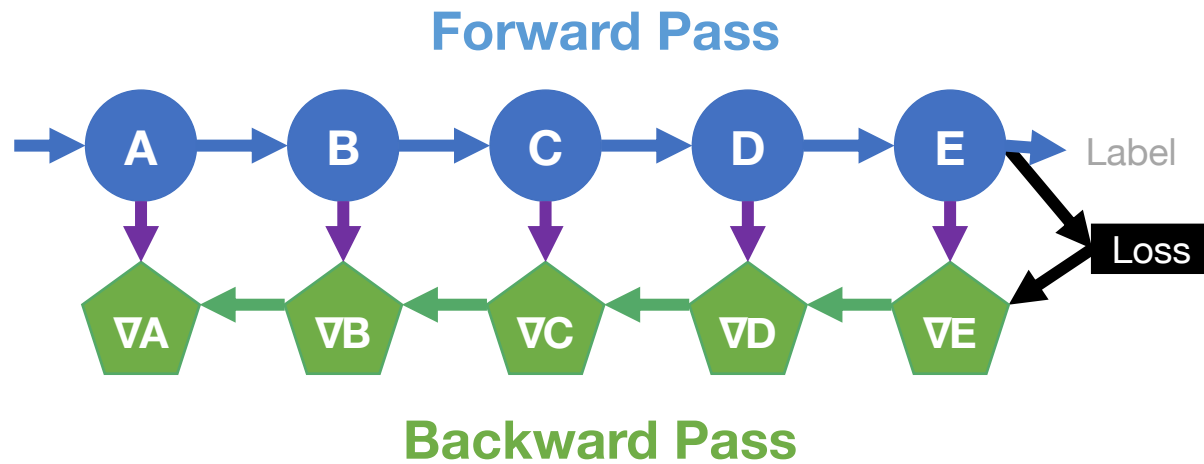
# RAM-optimized backpropagation policy

Recompute all layers as needed



# RAM-optimized backpropagation policy

Recompute **all layers**

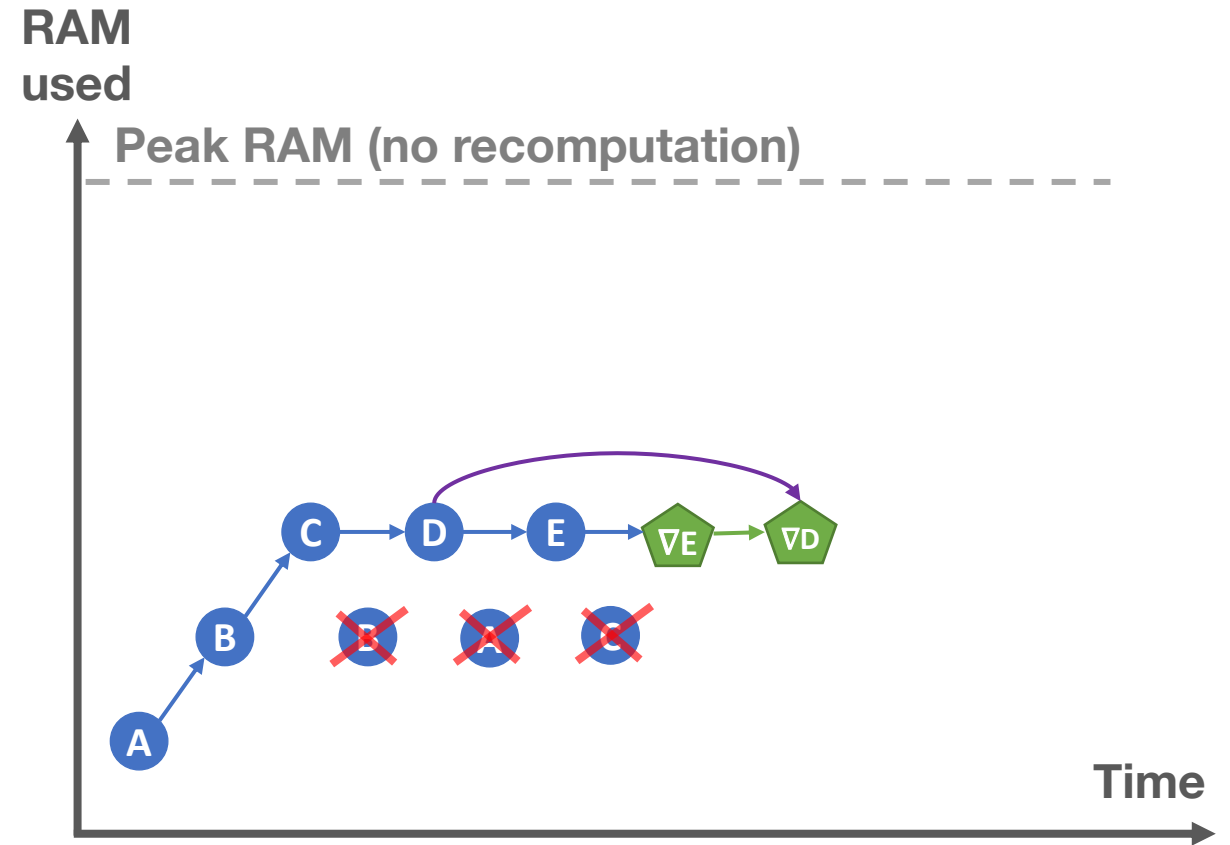
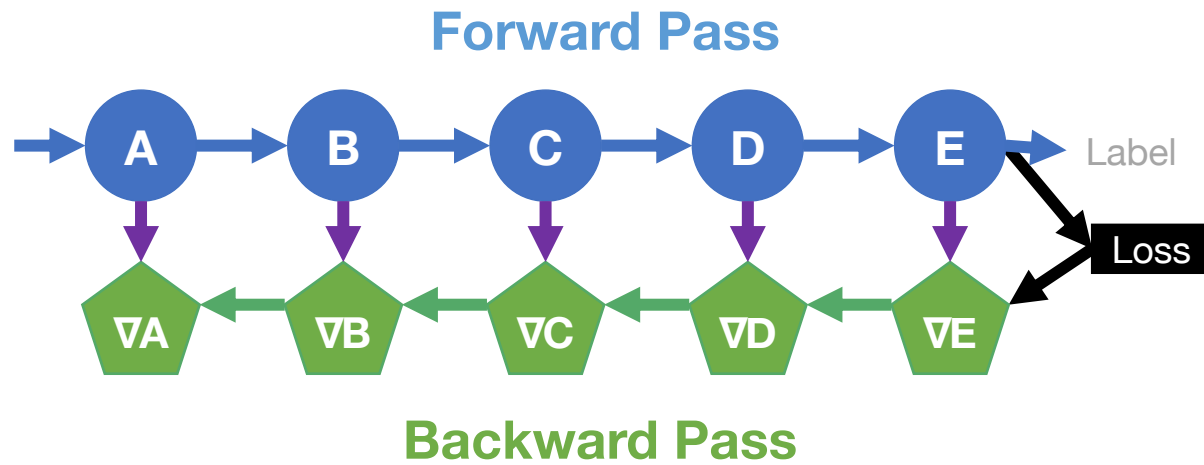


## How can we use less memory?

Free early & recompute

# RAM-optimized backpropagation policy

Recompute **all layers**

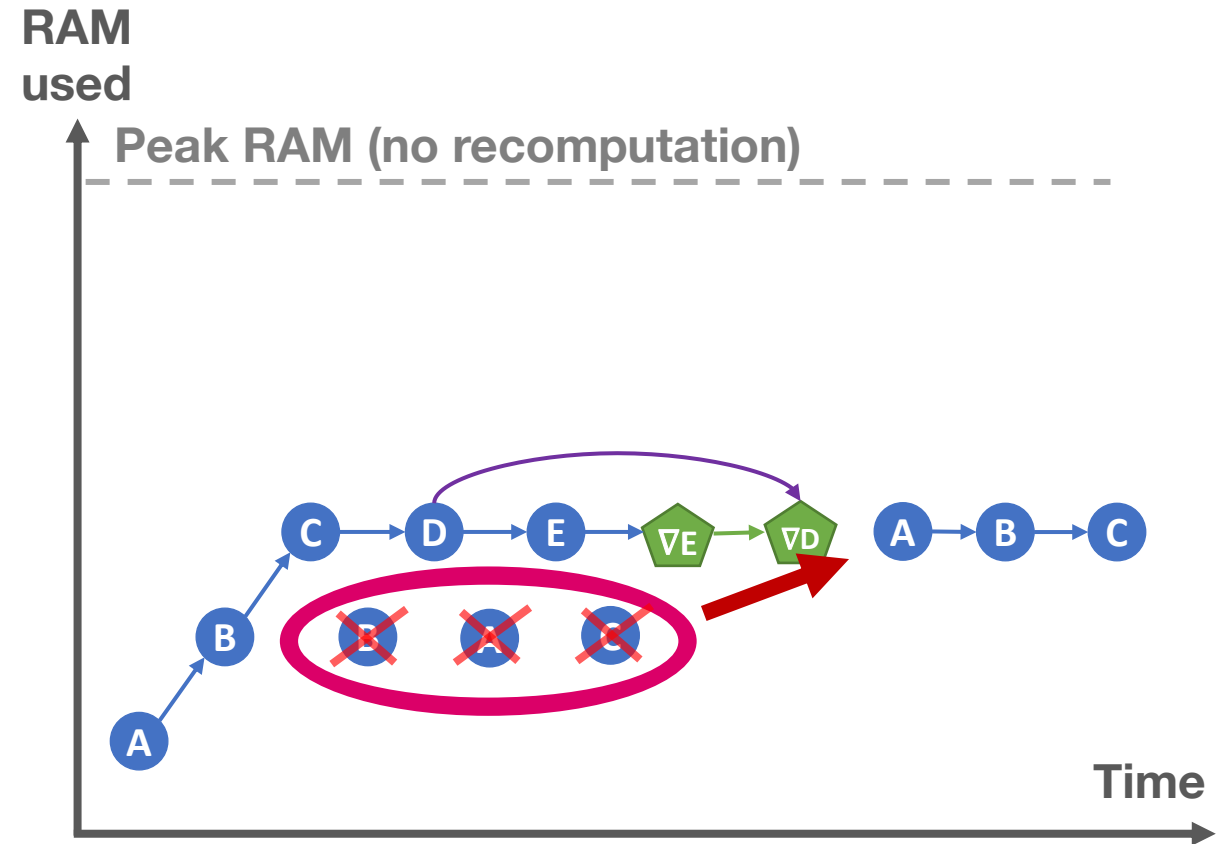
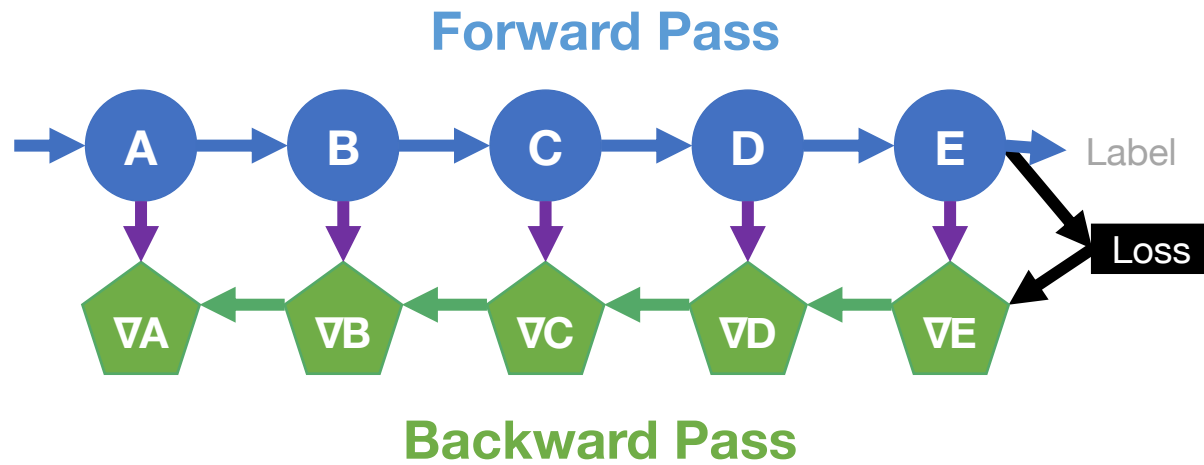


## How can we use less memory?

Free early & recompute

# RAM-optimized backpropagation policy

Recompute **all layers**

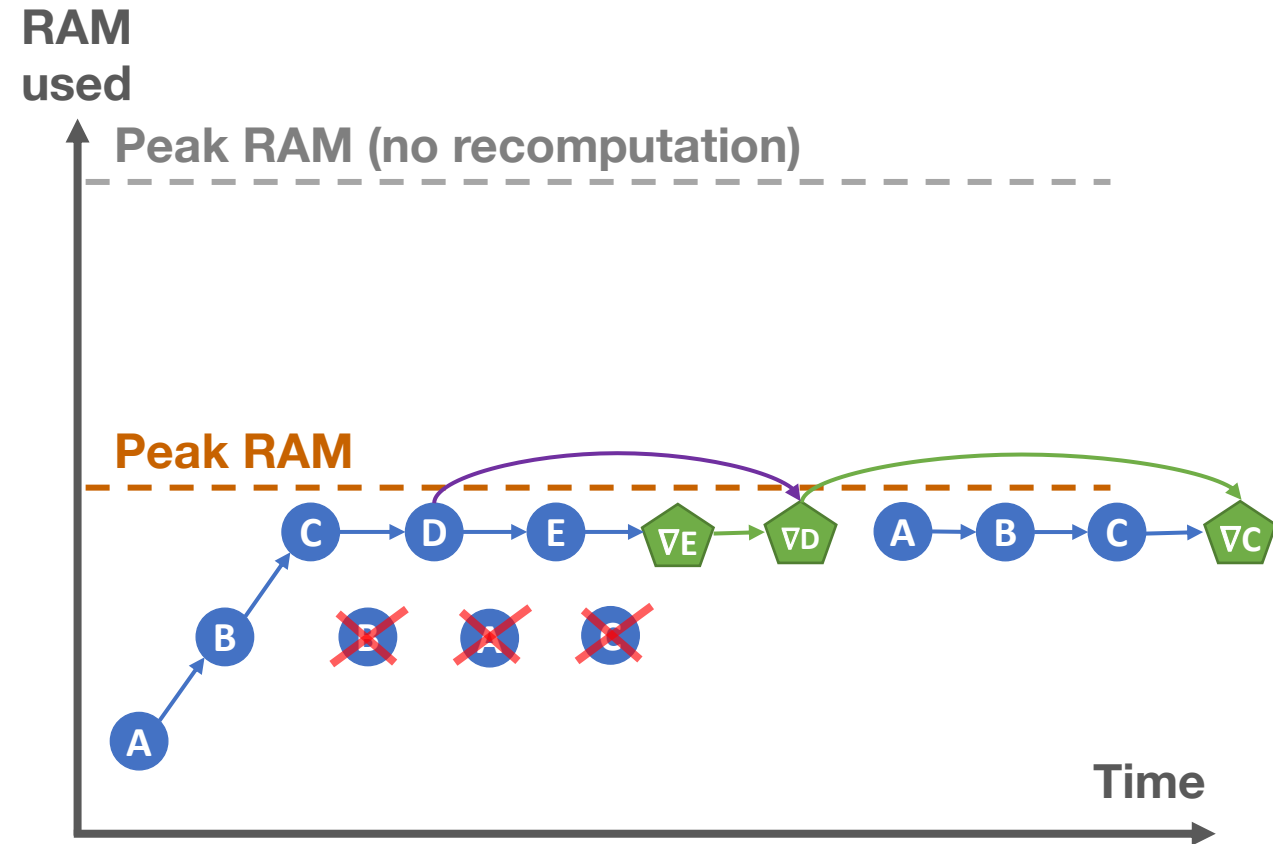
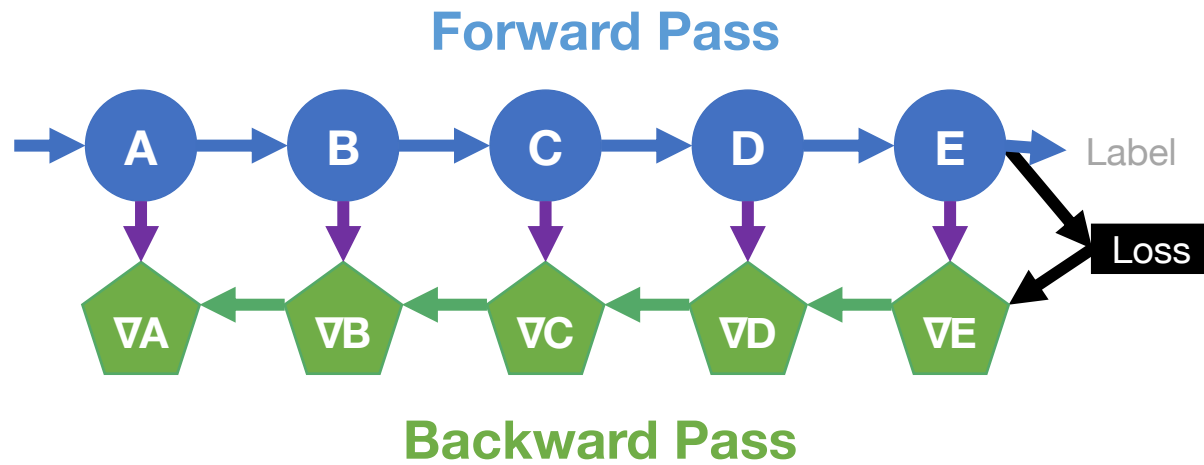


## How can we use less memory?

Free early & recompute

# RAM-optimized backpropagation policy

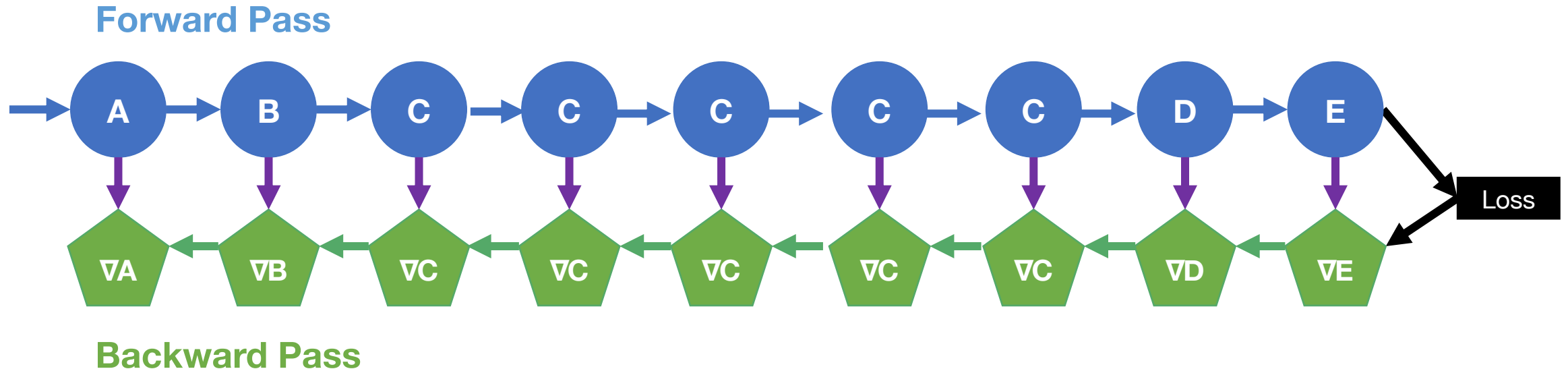
Recompute **all layers**



## How can we use less memory?

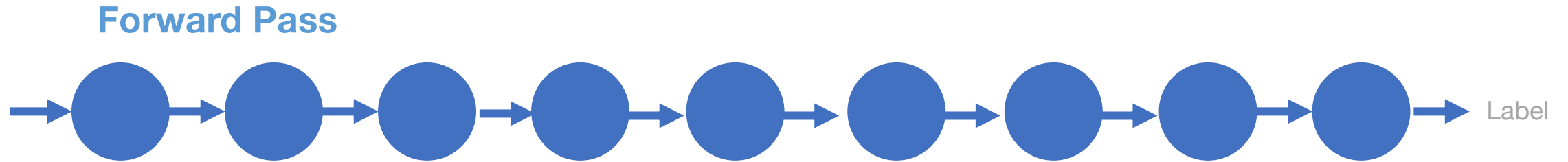
Free early & recompute

# How to choose which layers to recompute?

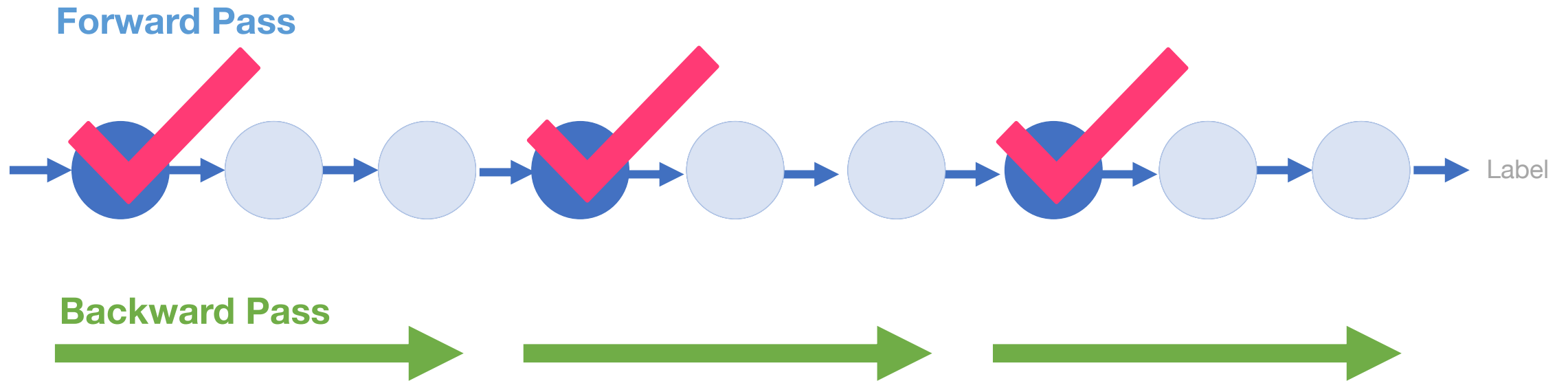




# How to choose which layers to recompute?

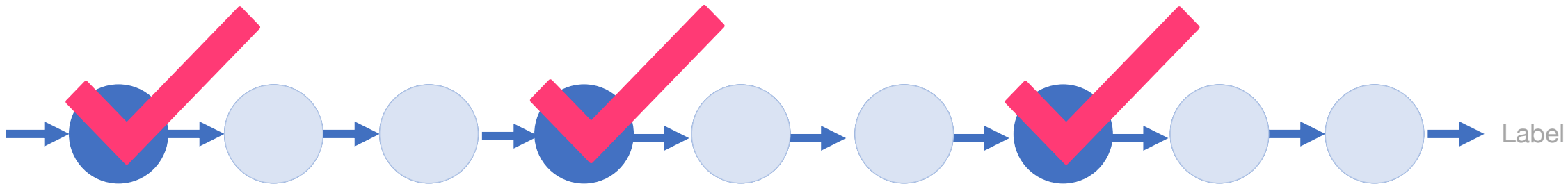


# How to choose which layers to recompute?



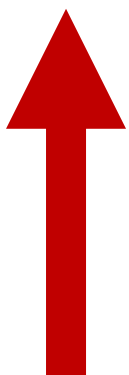
**Compute:**  $O(n)$  additional overhead

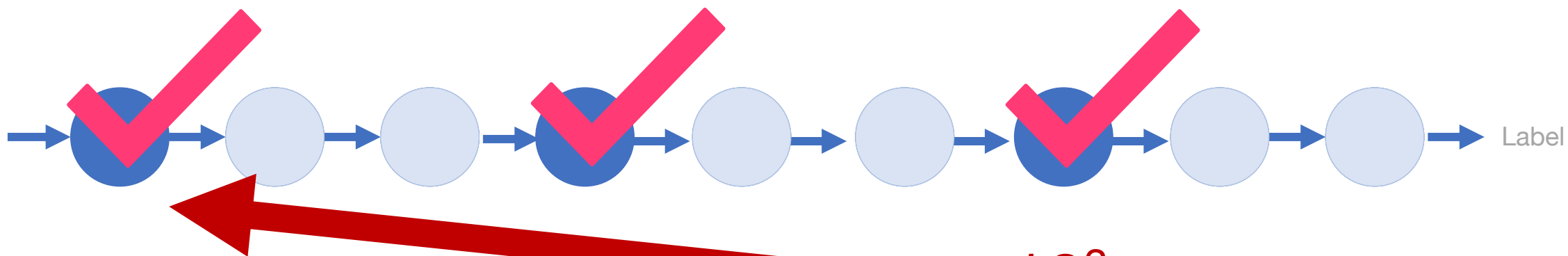
**RAM:**  $O(\sqrt{n})$  RAM usage



## Challenges of heuristics:

1. Variable runtime per layer

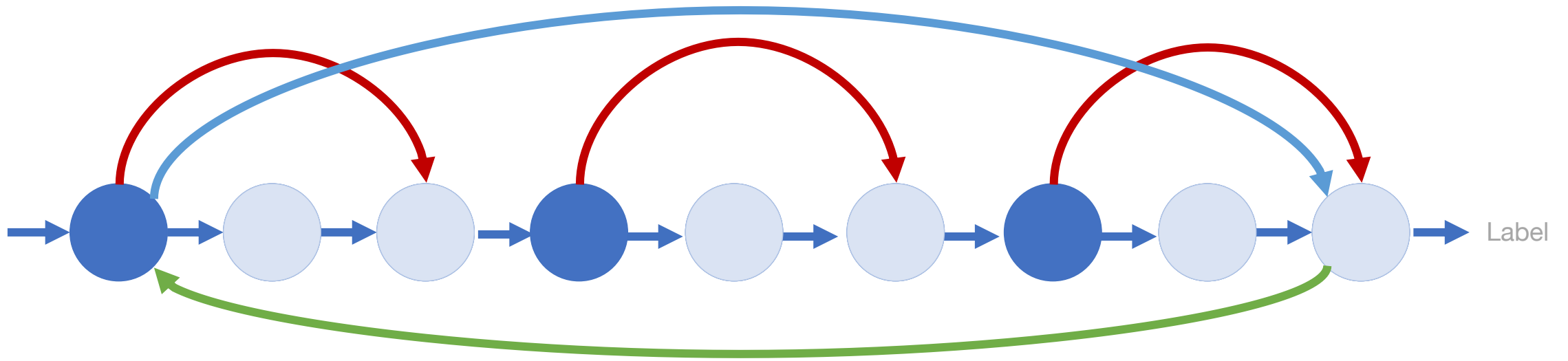
  
 $10^6\times$   
slower



## Challenges of heuristics:

1. Variable runtime per layer

2. Variable RAM usage per layer



## Challenges of heuristics:

1. Variable runtime per layer

2. Variable RAM usage per layer

3. Real DNNs are non-linear

# Prior work is suboptimal in general setting!

## Greedy heuristic

[Chen 2016]

[XLA authors 2017, 2020]

## Divide-and-conquer heuristic

[Griewank 2000]

[Kowarz 2006]

[Siskind 2018]

[Kumar 2019]

## Optimal for specific architecture

[Gruslys 2016]

[Feng 2018]

[Beaumont 2019]

## Challenges:

1. Variable runtime per layer

2. Variable RAM usage per layer

3. Real DNNs are non-linear

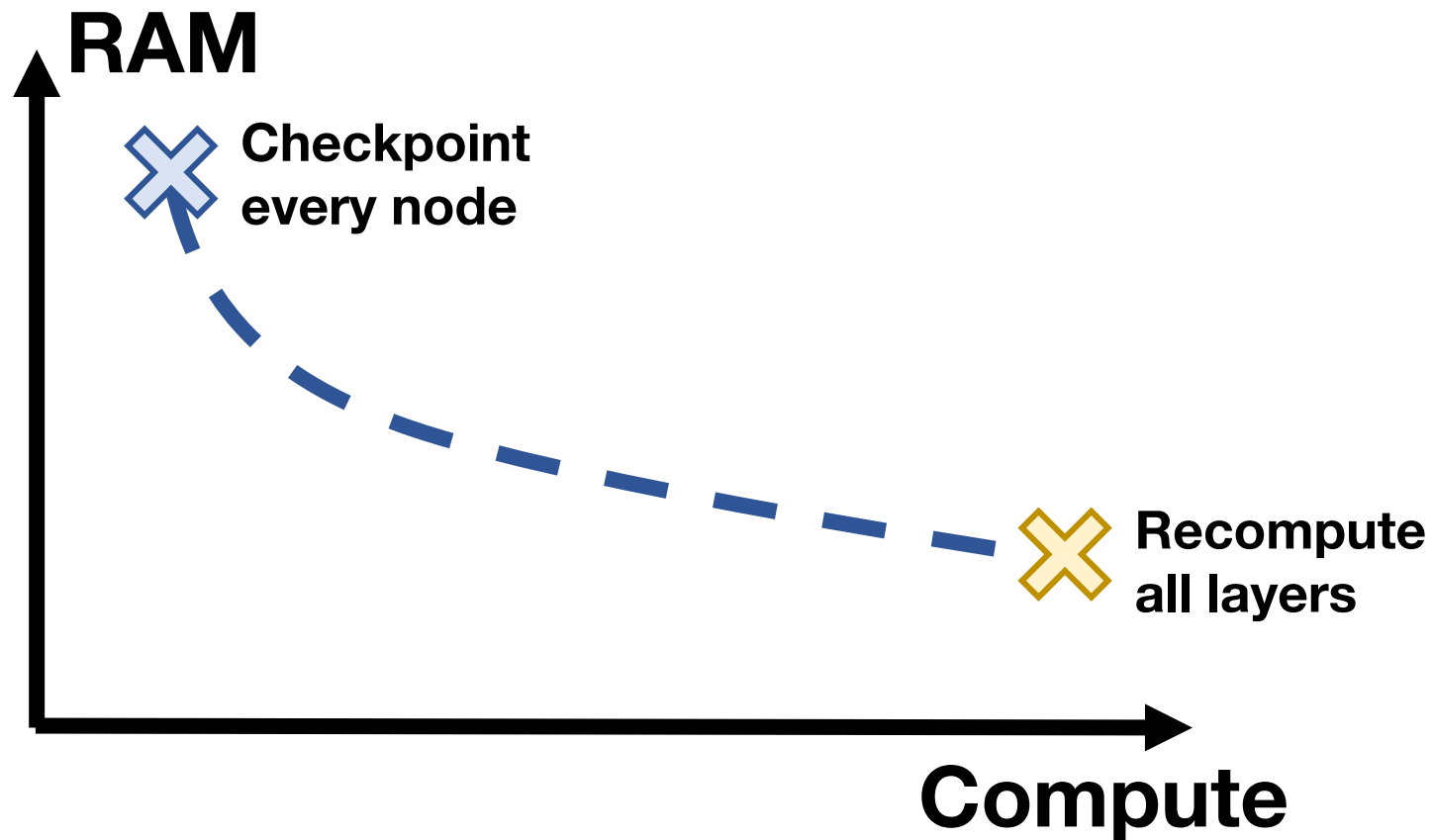
# Can we **optimally** trade-off RAM for compute?

*Let's be:*

1. Hardware-aware

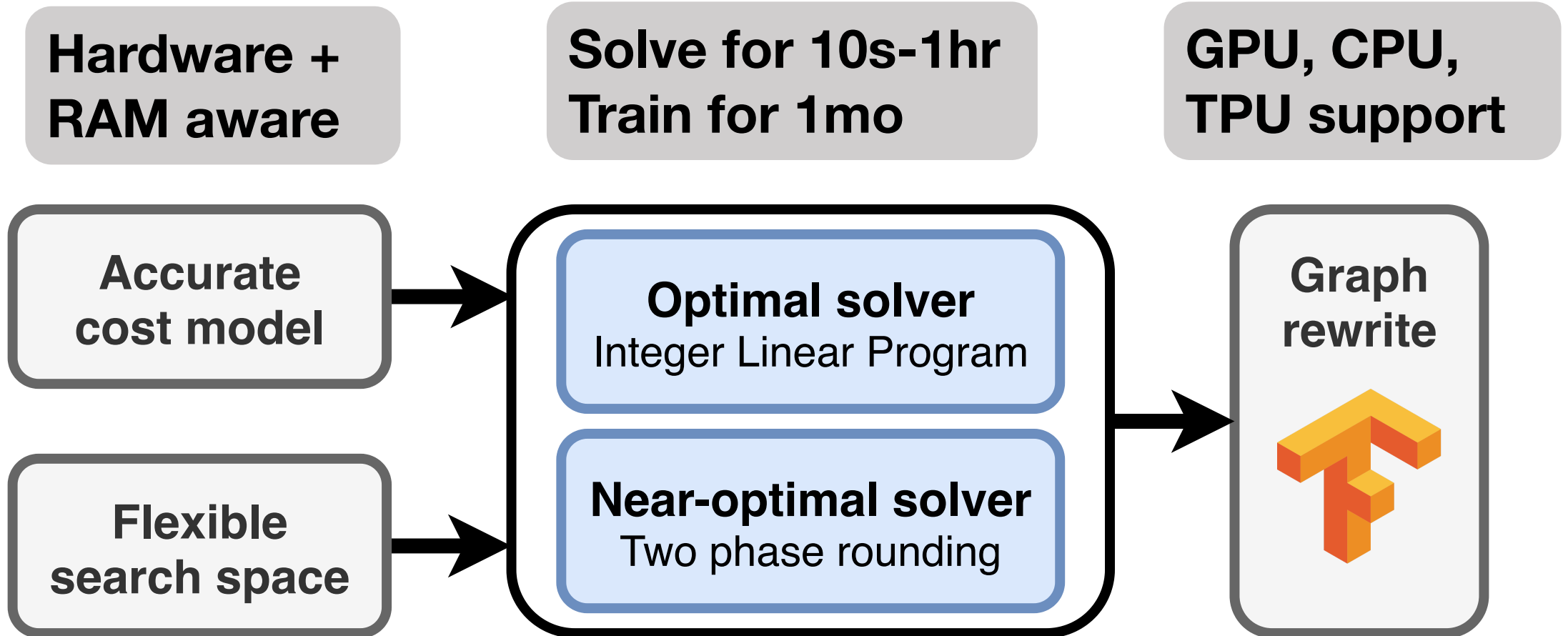
2. RAM-aware

3. DAG flexibility



# Checkmate

A system for **optimal** tensor rematerialization





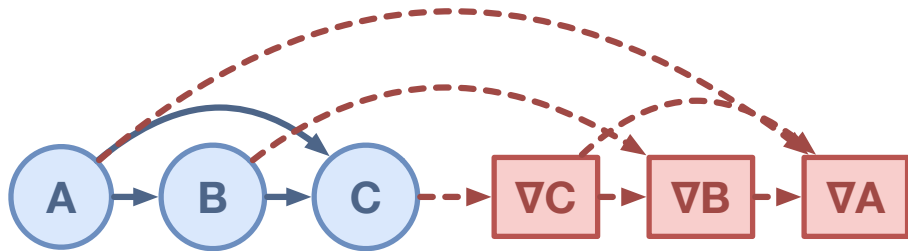
# Checkmate

A system for optimal tensor rematerialization



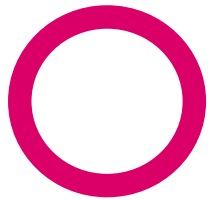
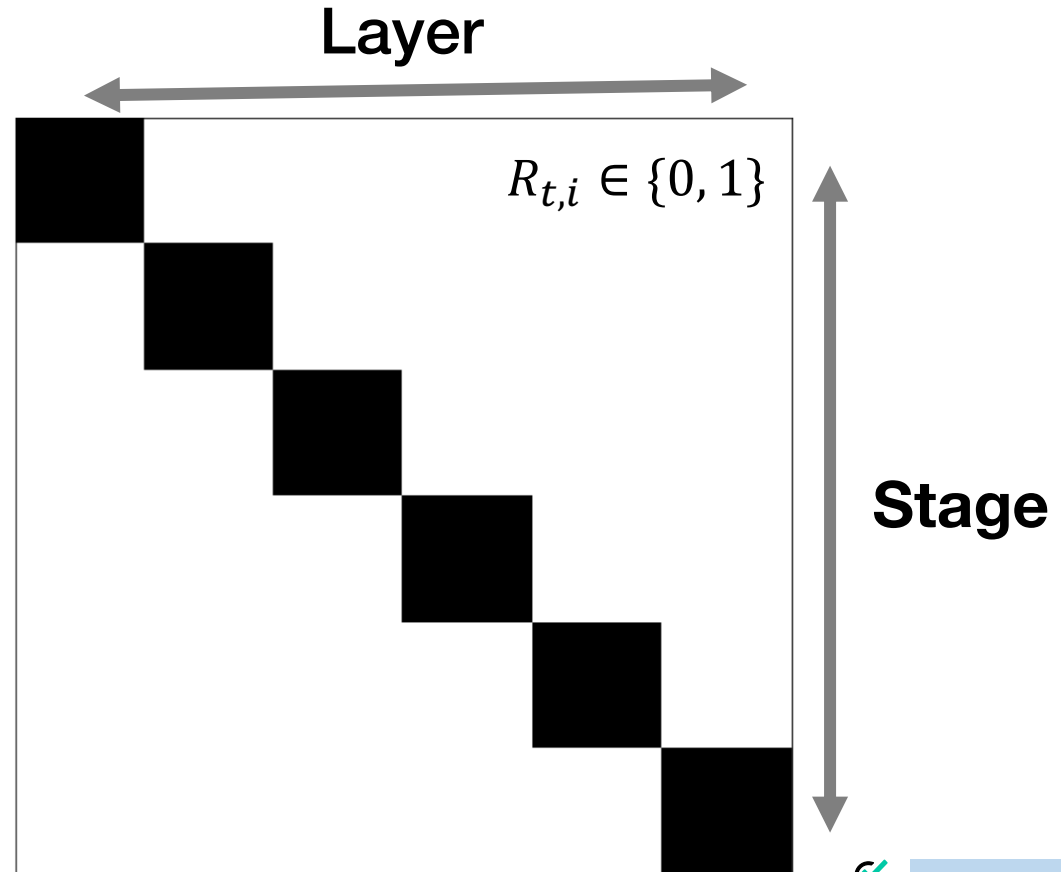
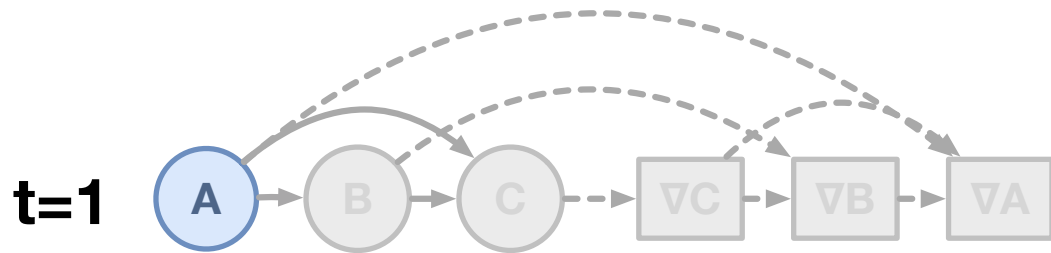
# Checkmate

A system for optimal tensor rematerialization



# Checkmate

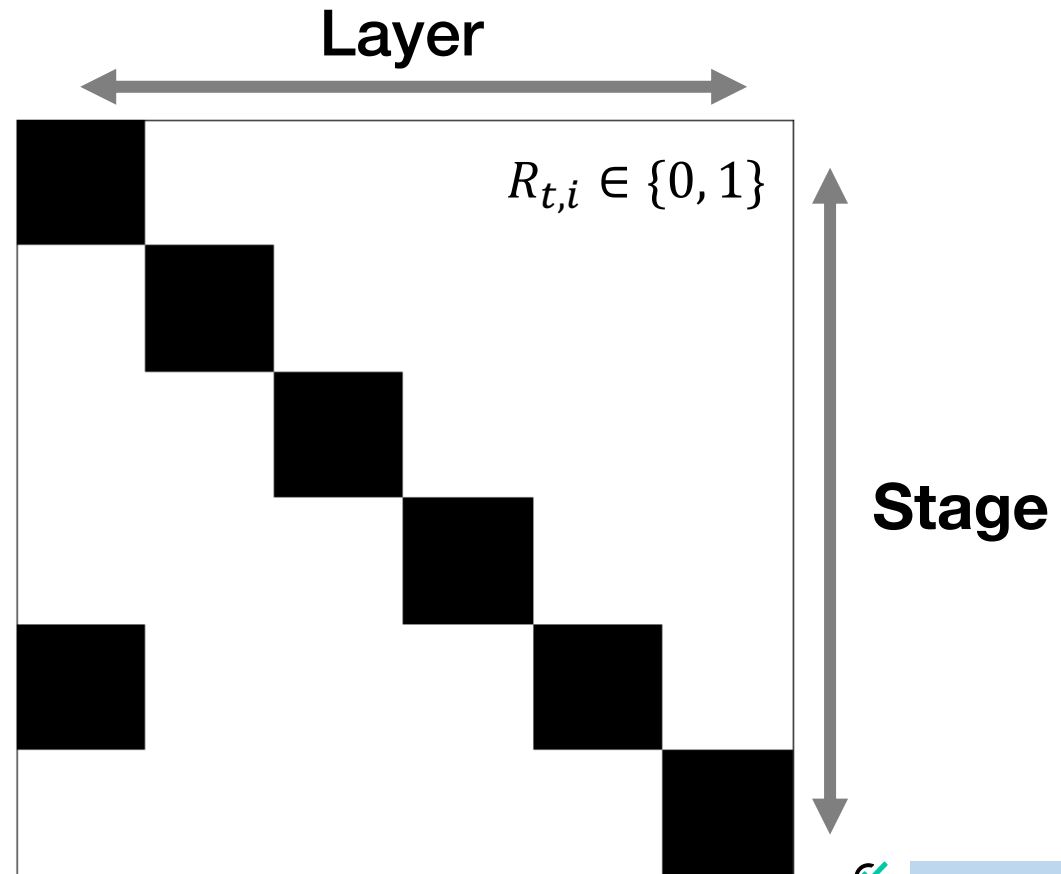
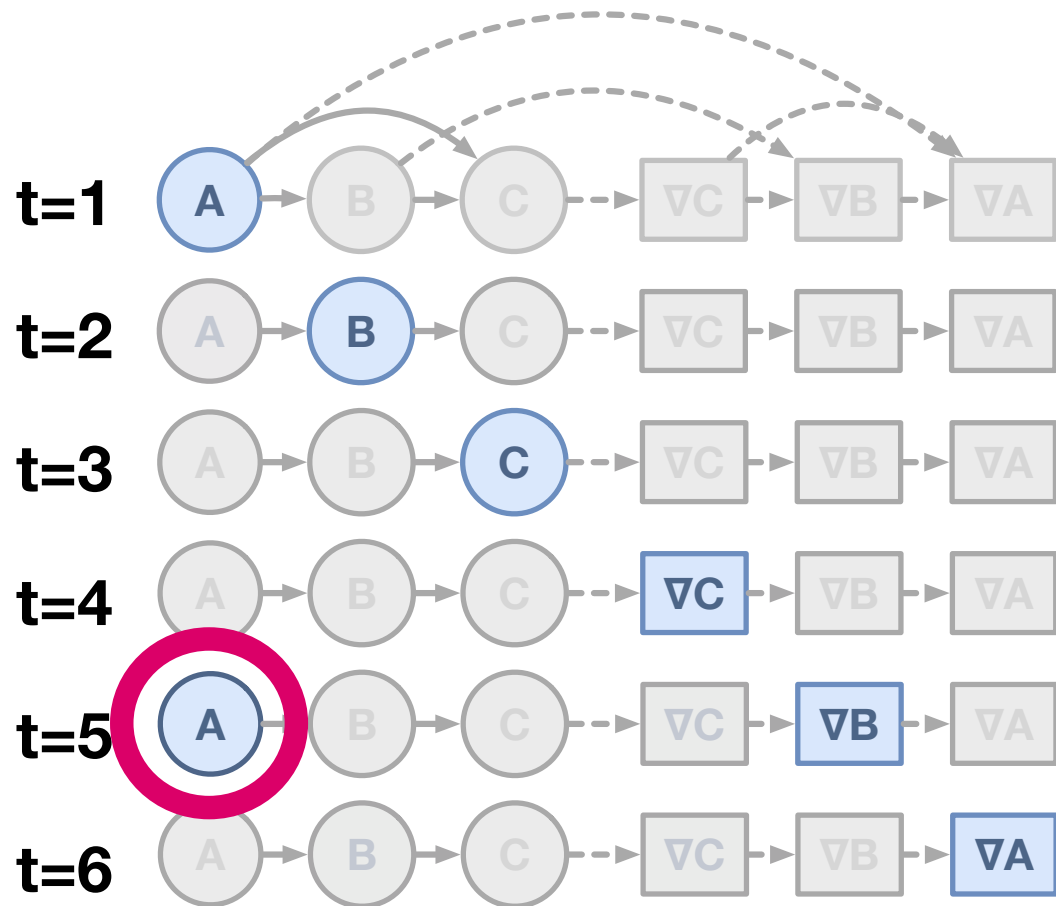
A system for **optimal** tensor rematerialization

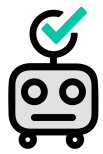




# Checkmate

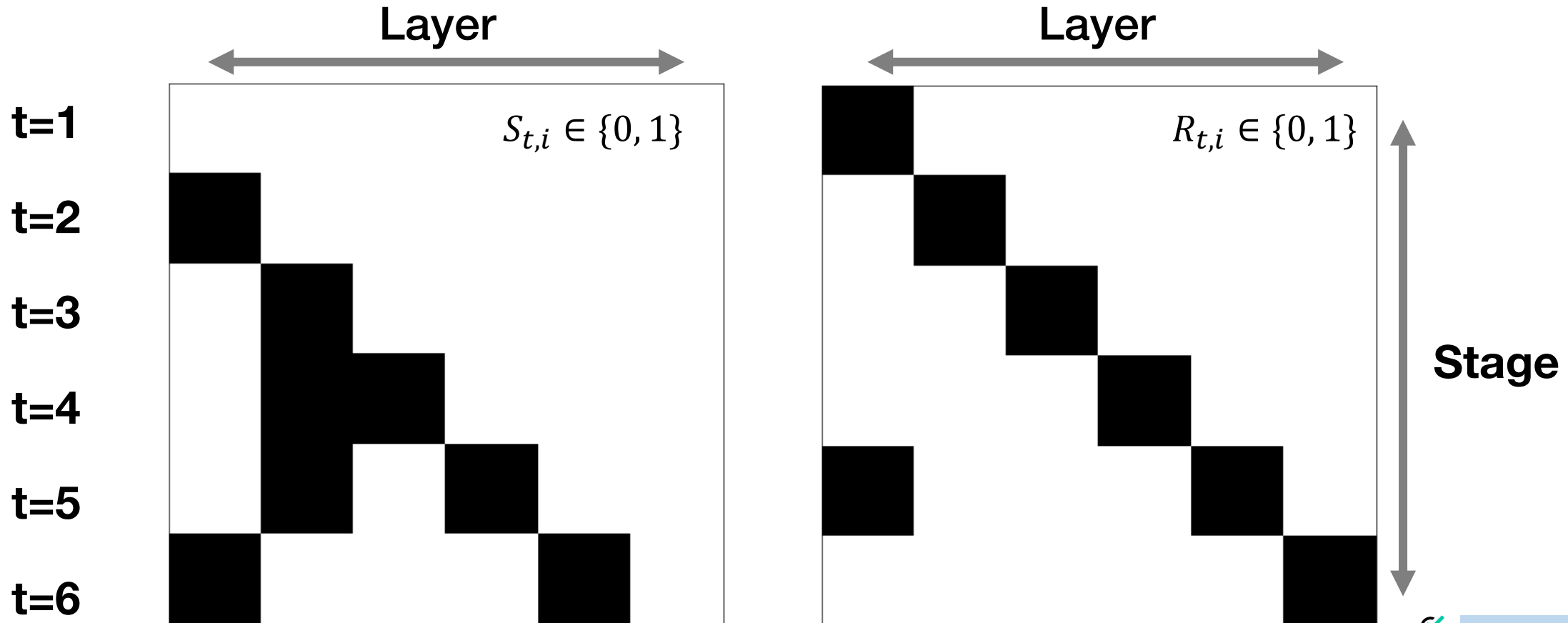
A system for optimal tensor rematerialization





# Checkmate

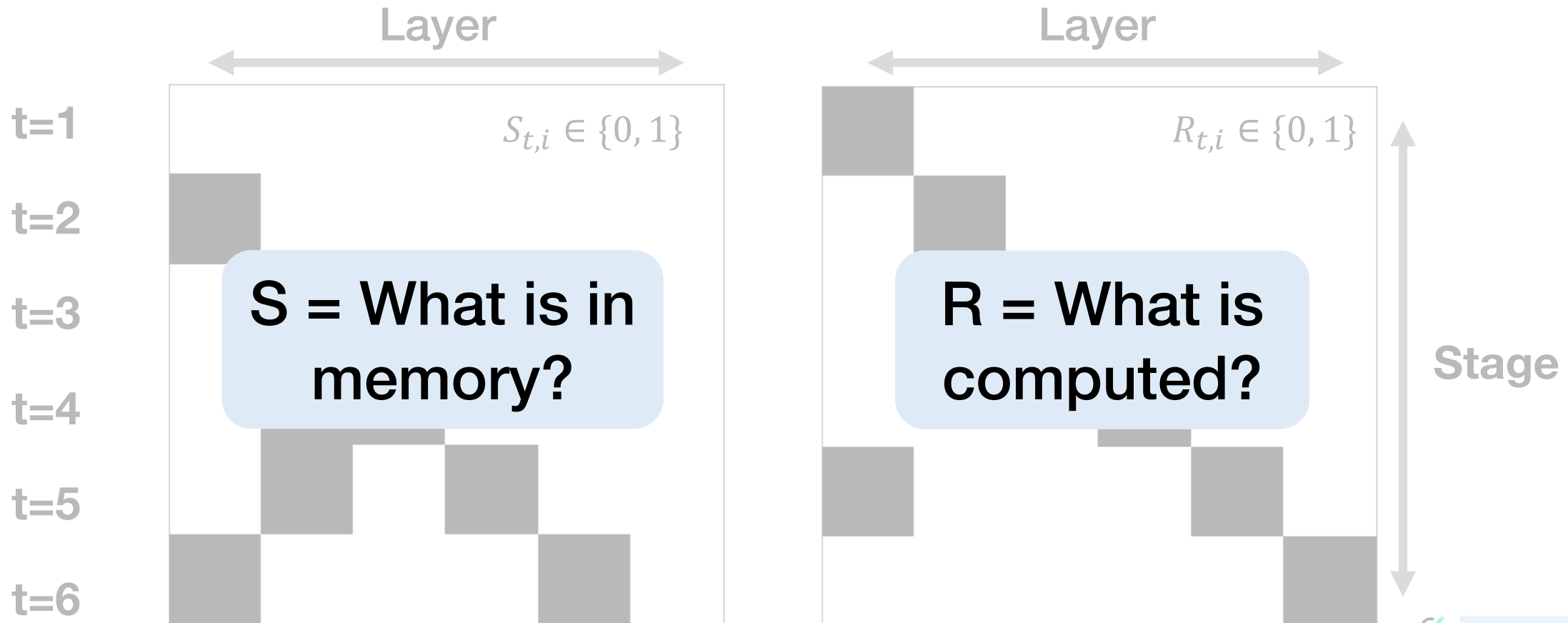
A system for optimal tensor rematerialization





# Checkmate

A system for optimal tensor rematerialization

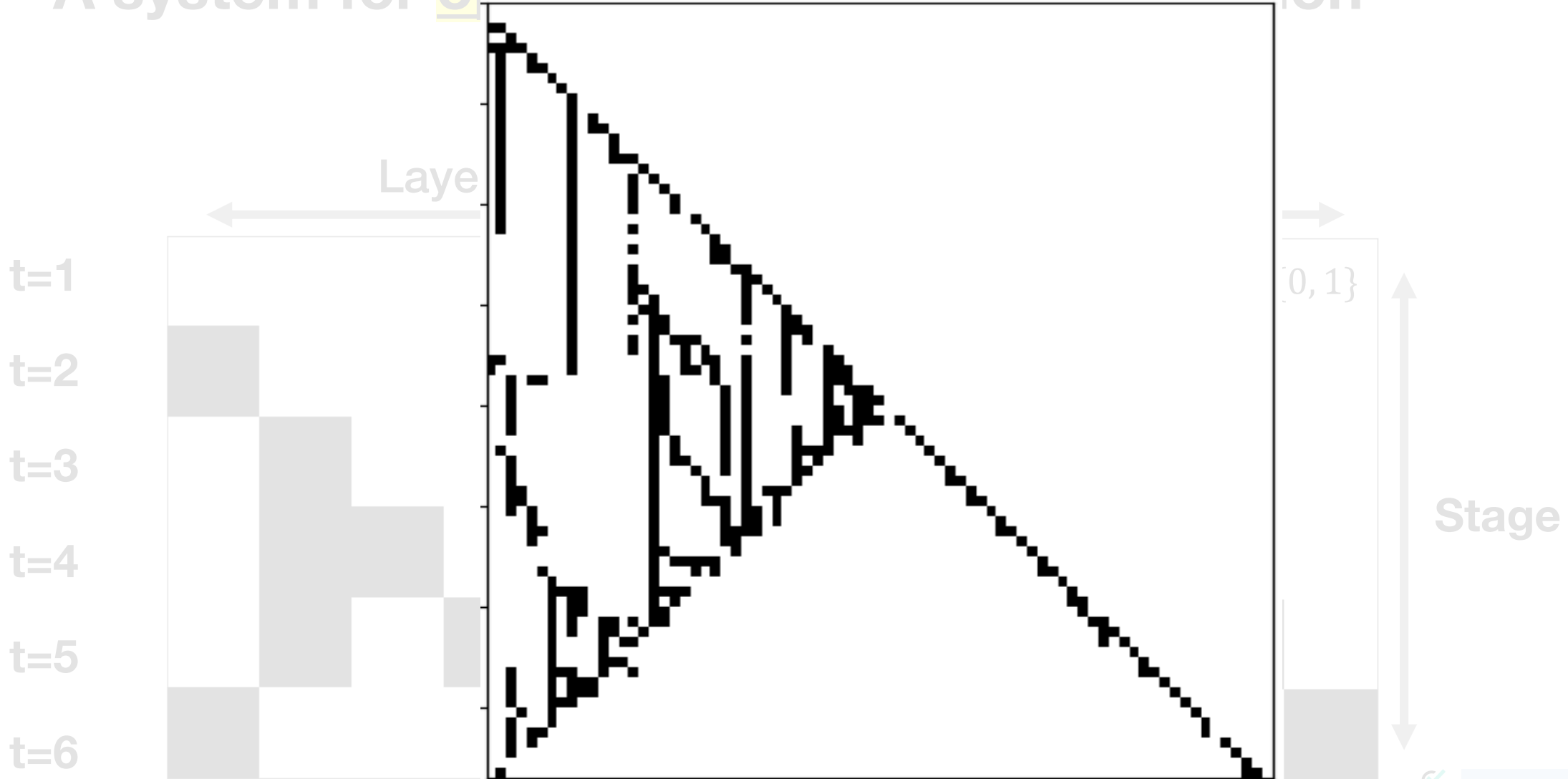




# Checkmate AI

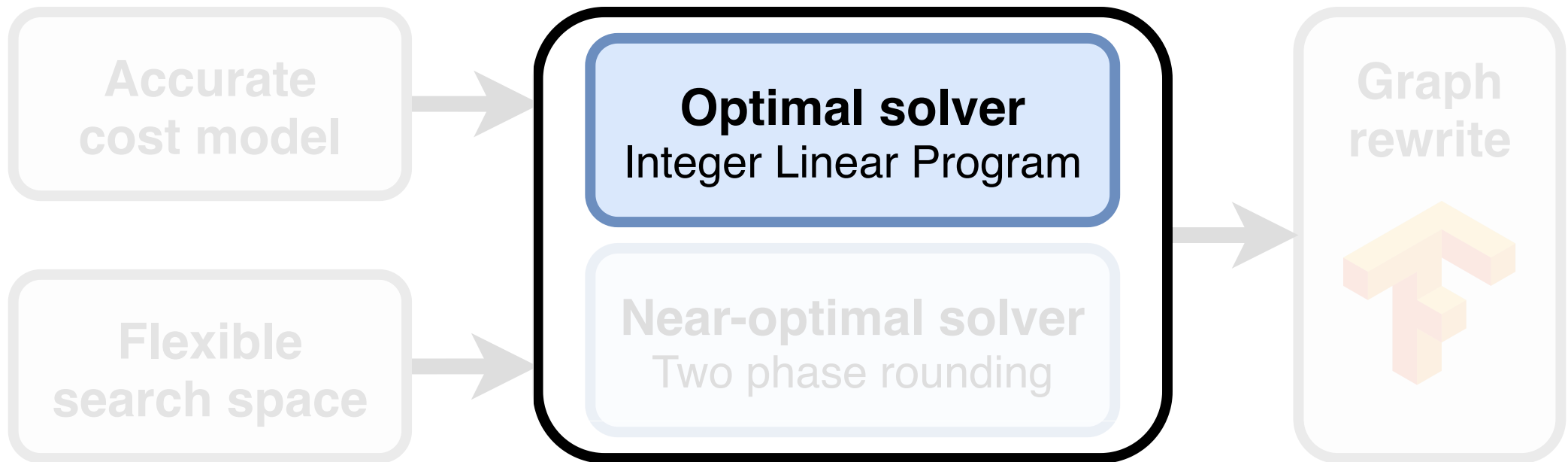
## Example of optimal “S” (SegNet)

A system for optimal tensor rematerialization

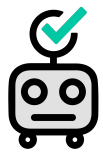


# Checkmate

A system for optimal tensor rematerialization

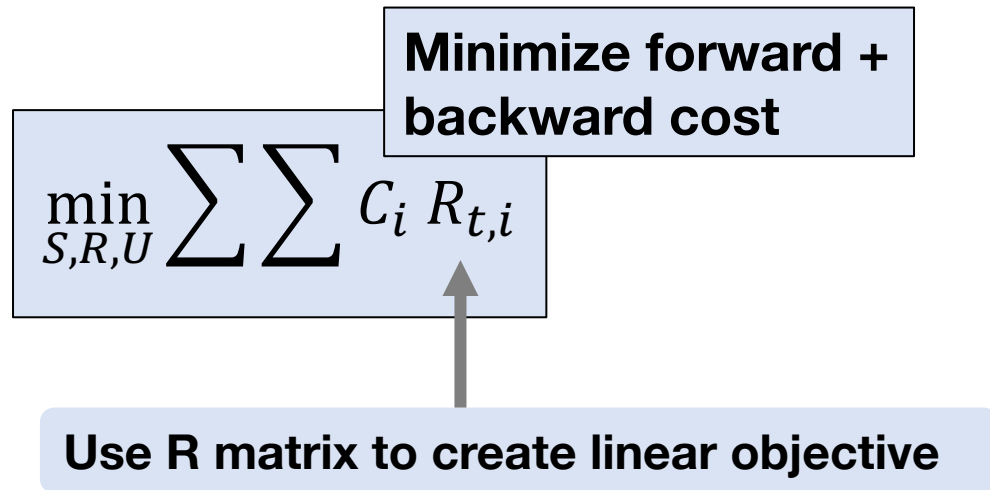






# Checkmate

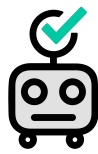
A system for optimal tensor rematerialization



## Decision variables

- $S_{t,i} \in \{0, 1\}$  Layer  $i$  stored for stage  $t$
- $R_{t,i} \in \{0, 1\}$  Layer  $i$  (re)computed in stage  $t$





# Checkmate

A system for optimal tensor rematerialization

Minimize forward +  
backward cost

$$\min_{S,R,U} \sum \sum C_i R_{t,i}$$

**Decision variables**

$S_{t,i} \in \{0, 1\}$  Layer  $i$  stored for stage  $t$

$R_{t,i} \in \{0, 1\}$  Layer  $i$  (re)computed in stage  $t$

**Correctness**

$$R_{t,j} \leq R_{t,i} + S_{t,i}$$

$$S_{t,i} \leq R_{t-1,i} + S_{t-1,i}$$

“A layer’s dependencies must  
be computed before evaluation”

“A layer must be computed  
before it can be stored in RAM”





# Checkmate

A system for optimal tensor rematerialization

Minimize forward +  
backward cost

$$\min_{S,R,U} \sum \sum C_i R_{t,i}$$

Correctness

$$R_{t,j} \leq R_{t,i} + S_{t,i}$$

$$S_{t,i} \leq R_{t-1,i} + S_{t-1,i}$$

Memory limit

$$U_{t,k} \leq \text{budget}, \dots$$

Constrain memory via an implicit variable  
to model memory usage at each stage

Decision variables

$S_{t,i} \in \{0, 1\}$  Layer  $i$  stored for stage  $t$

$R_{t,i} \in \{0, 1\}$  Layer  $i$  (re)computed in stage  $t$

$U_{t,i} \in \mathbb{R}_+$  Memory usage in stage  $t$





# Checkmate

## A system for optimal tensors

Minimize forward + backward cost

$$\min_{S,R,U} \sum \sum C_i R_{t,i}$$

Correctness

$$R_{t,j} \leq R_{t,i} + S_{t,i}$$

$$S_{t,i} \leq R_{t-1,i} + S_{t-1,i}$$

Memory limit

$$U_{t,k} \leq \text{budget}, \dots$$

Memory accounting details in paper

Constrain me to model memory usage at each stage

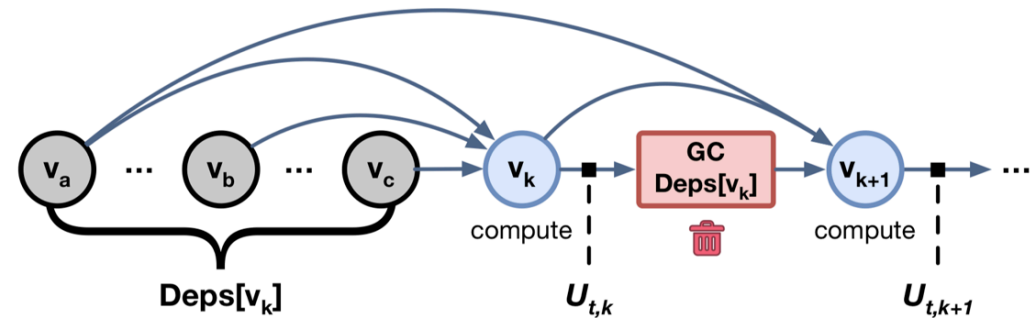
Constrain memory at all execution steps

$$U_{t,k} \leq \text{budget}$$

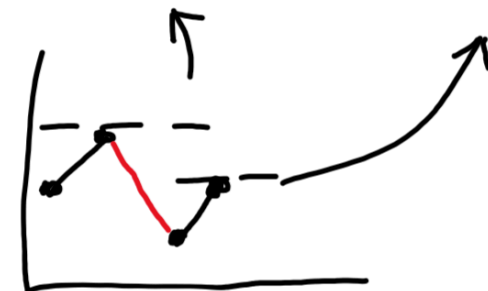
More checkpoints  $\rightarrow$  more initial memory

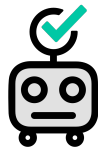
$$U_{t,0} = \sum M_i S_{t,i}$$

$$U_{t,k+1} = U_{t,k} - \text{GC Deps}[v_k] + M_{k+1} R_{t,k+1}$$



Memory Usage





# Checkmate

A system for optimal tensor rematerialization

Minimize forward +  
backward cost

$$\min_{S,R,U} \sum \sum C_i R_{t,i}$$

Correctness

$$R_{t,j} \leq R_{t,i} + S_{t,i}$$

$$S_{t,i} \leq R_{t-1,i} + S_{t-1,i}$$

Memory limit

$$U_{t,k} \leq \text{budget}, \dots$$

How long is the solve time?

9 hours 🤔



# Checkmate

A system for **optimal** tensor rematerialization

Minimize forward +  
backward cost

$$\min_{S,R,U} \sum \sum C_i R_{t,i}$$

Correctness

$$R_{t,j} \leq R_{t,i} + S_{t,i}$$

$$S_{t,i} \leq R_{t-1,i} + S_{t-1,i}$$

Memory limit

$$U_{t,k} \leq \text{budget}, \dots$$

Partition schedule into  
frontier-advancing stages  
9 hours  $\rightarrow$  0.2 seconds

Tractability

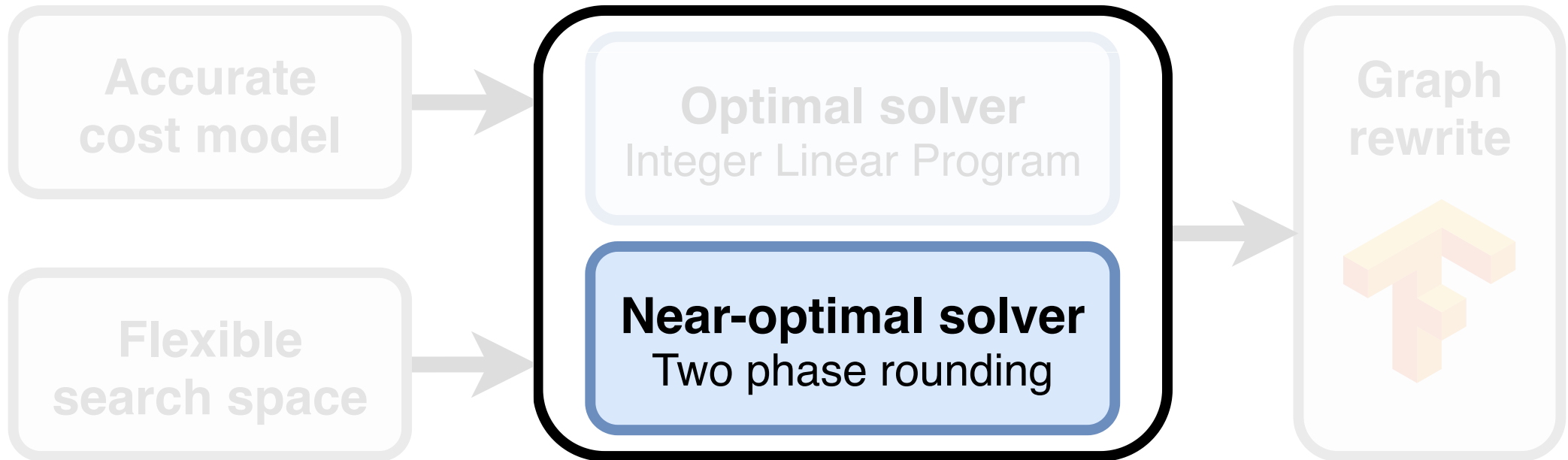
$$R_{t,t} = 1$$

$R, S, U$  lower triangular

Prunes  $n!$   
permutations  
of nodes

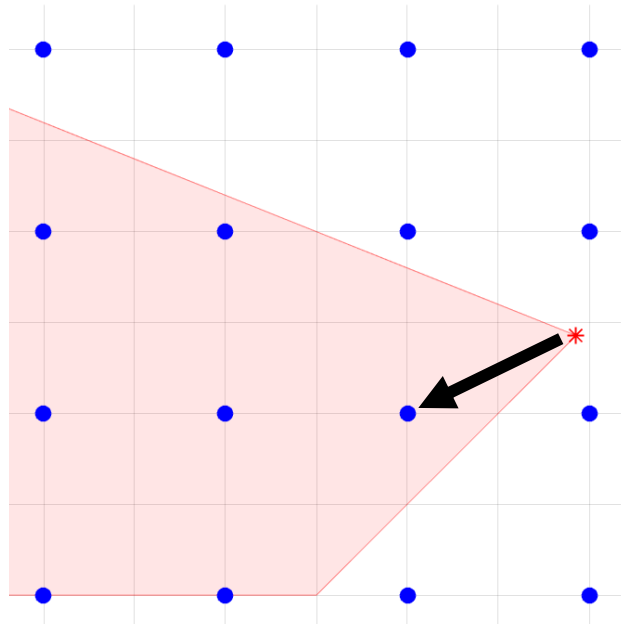
# Checkmate

A system for optimal tensor rematerialization



# ILP optimization is NP-hard (combinatorial search)

Polynomial-time approximation?



1. Relax boolean constraints
2. Solve LP
3. Round solution

**How to maintain feasibility?**

**Insight: Given  $S$ , optimal  $R$  easy to compute**

**Proposed method: Two-Phase Rounding**

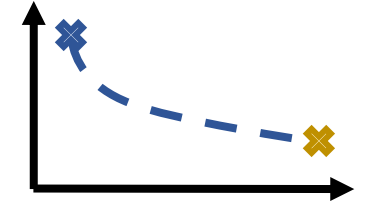
Round  $S$ , solve other variables optimally



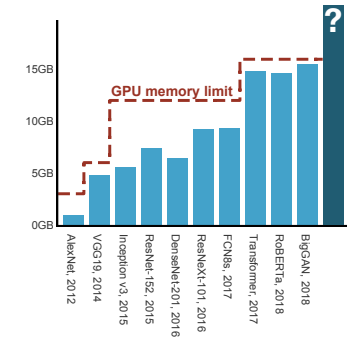


# Evaluation: Questions

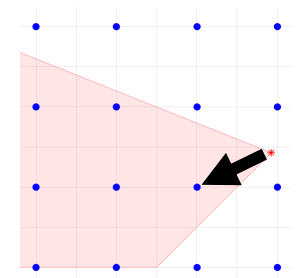
1. What is the memory vs compute trade-off?



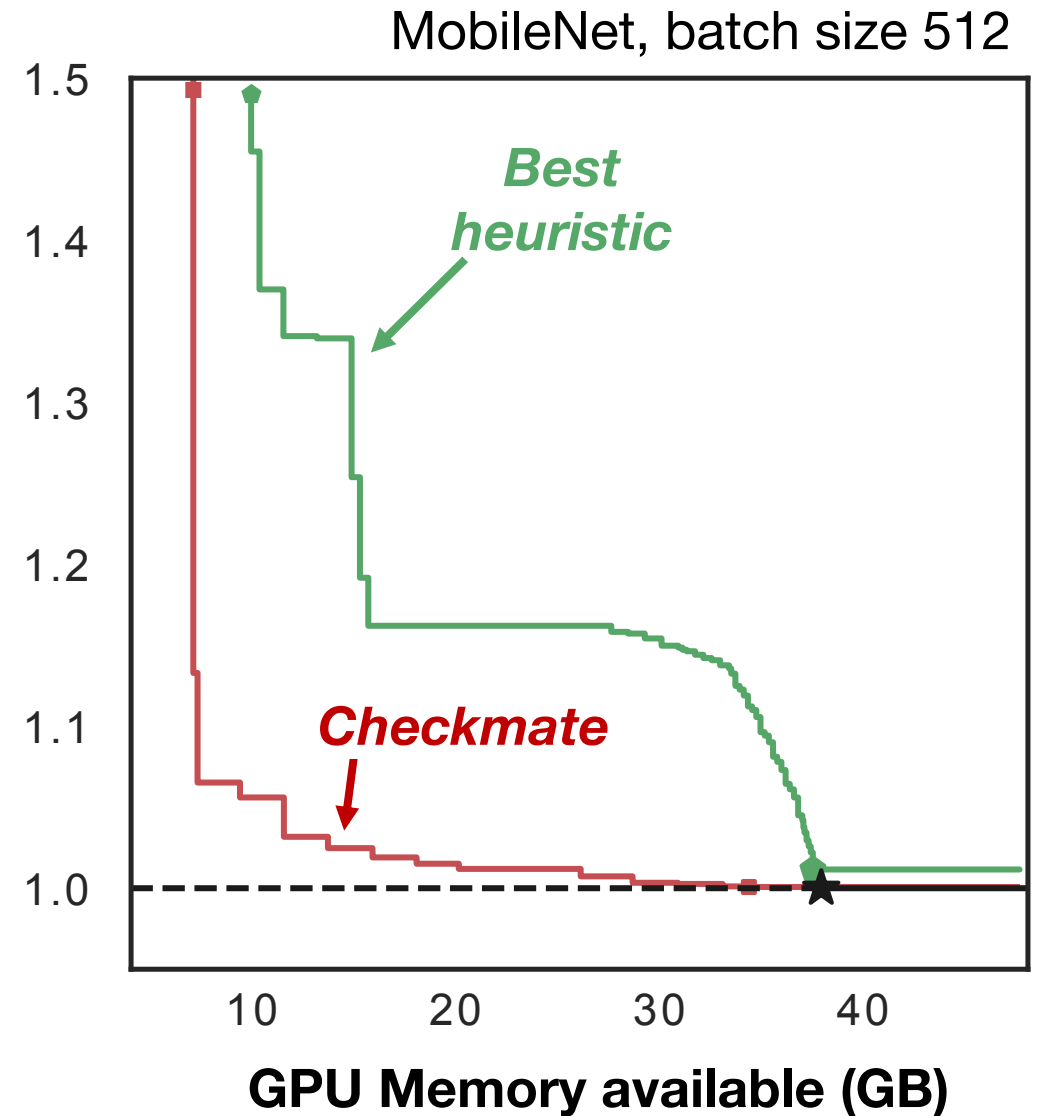
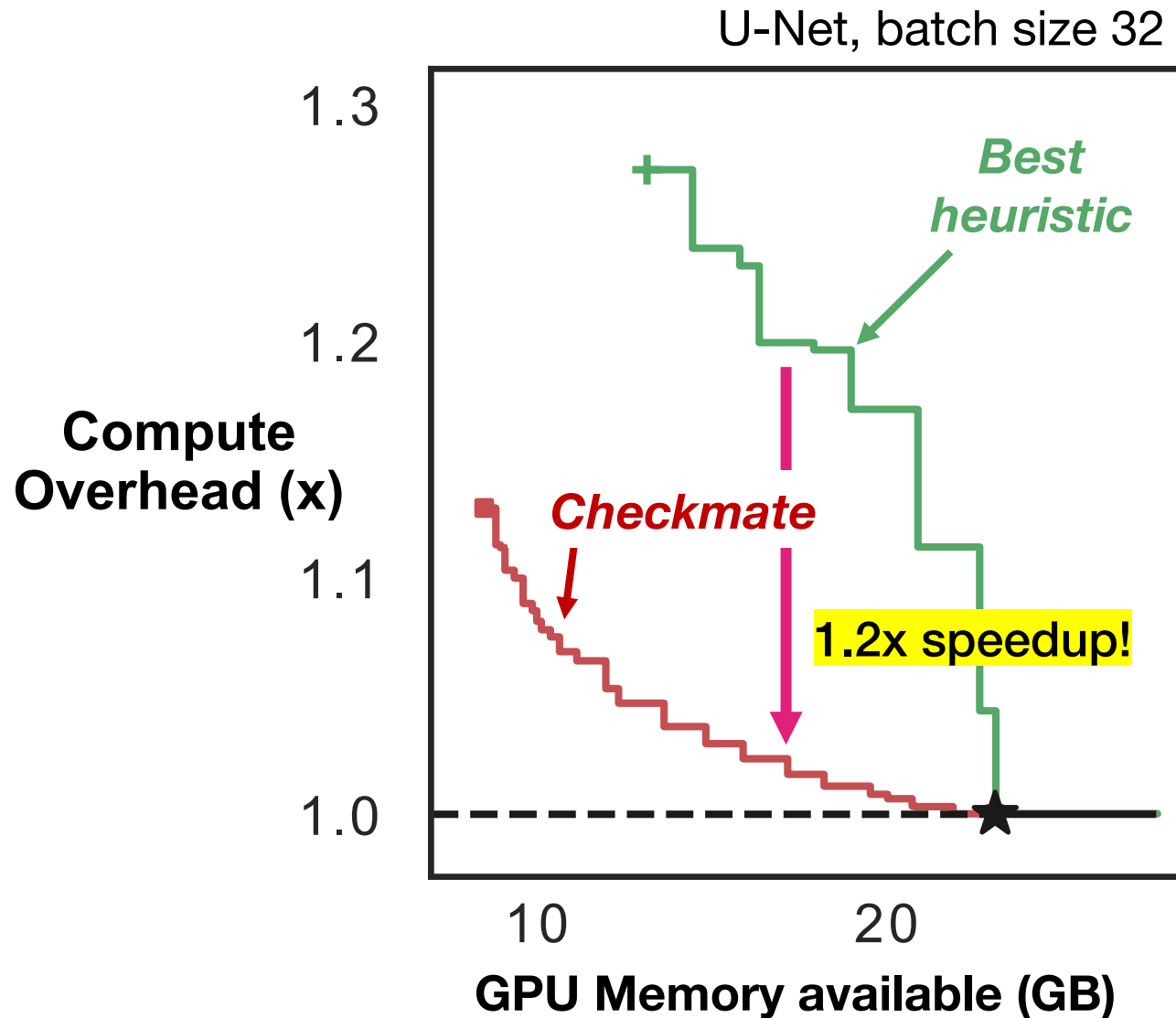
2. How much can we increase batch/model size?



3. How well does two-phase rounding do?

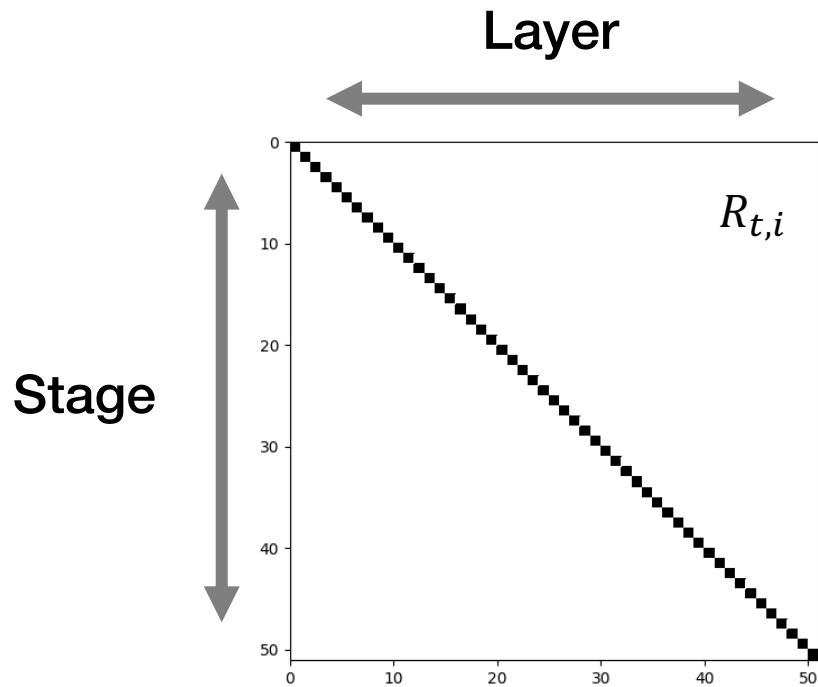


# Evaluation: What is the memory vs compute trade-off?

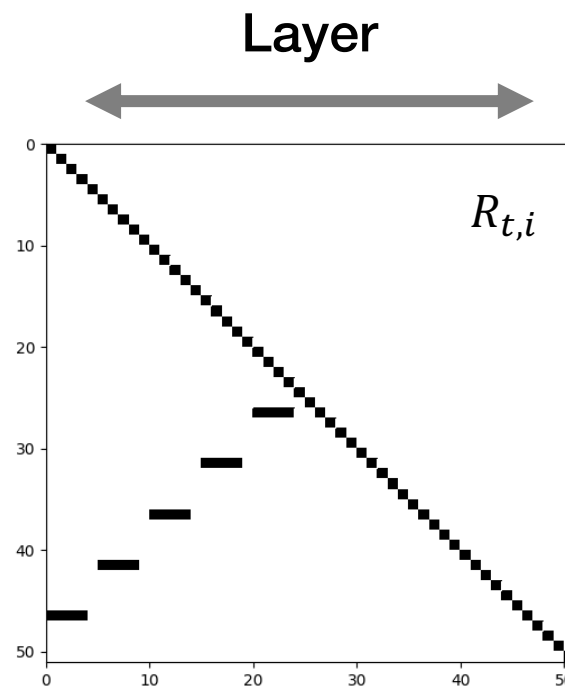


# Evaluation: How much can we increase batch size?

VGG19  
224x224 images

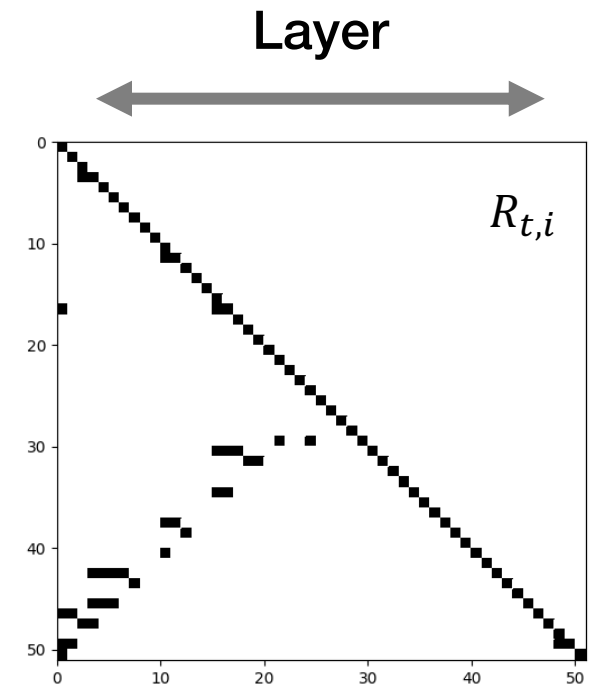


No rematerialization  
Batch size 167



Square root heuristic  
Batch size 197

1.18x larger!

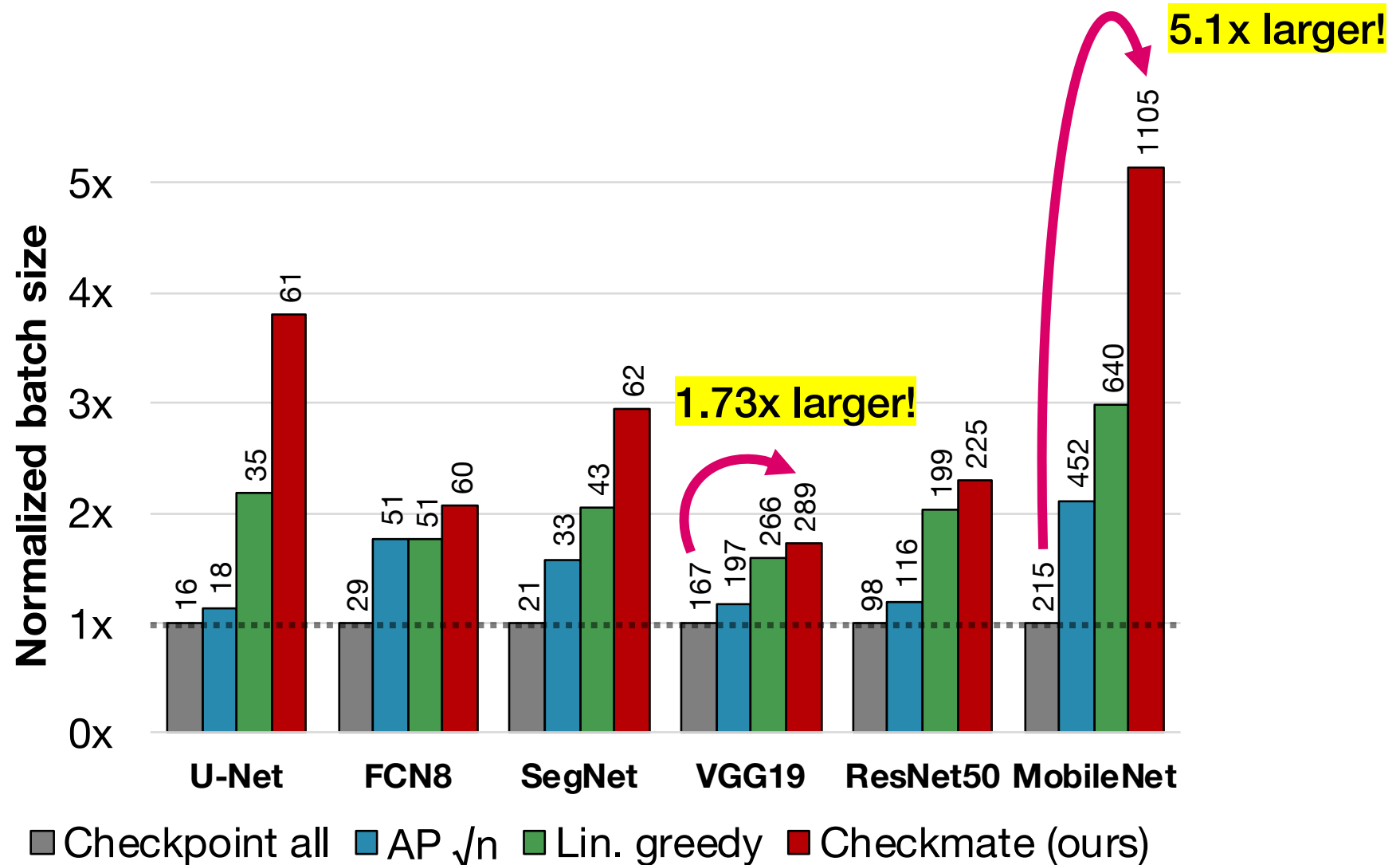


Checkmate  
Batch size 289

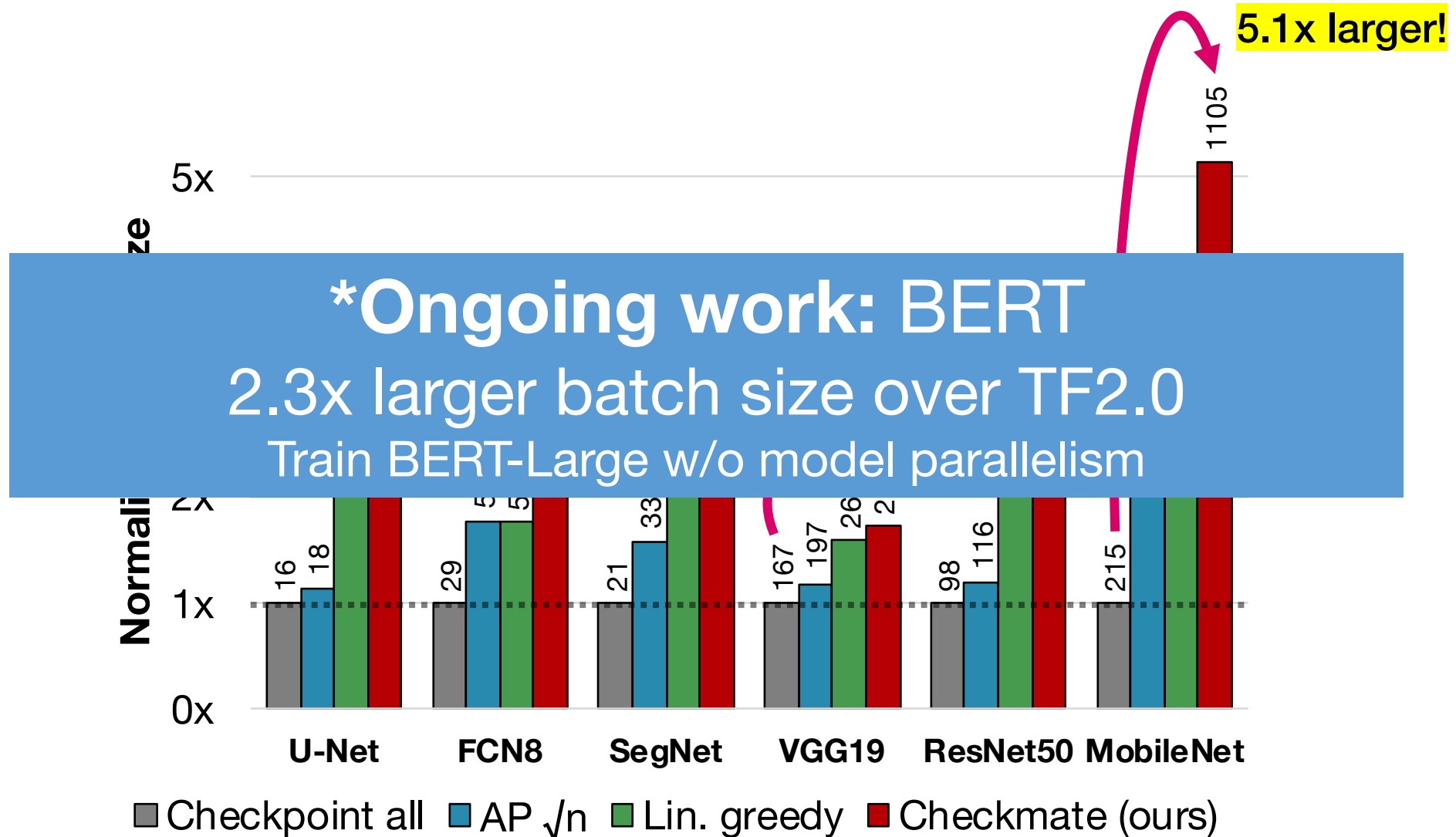
1.73x larger! 10 sec solve



# Evaluation: How much can we increase batch size?



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# Evaluation: How well does 2P rounding approximate ILP?

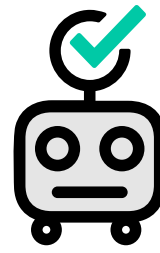
	AP $\sqrt{n}$	AP greedy	Griewank $\log n$	Two-phase LP rounding
MobileNet	1.14×	1.07×	7.07×	<b>1.06</b> ×
VGG16	1.28×	1.06×	1.44×	<b>1.01</b> ×
VGG19	1.54×	1.39×	1.75×	<b>1.00</b> ×
U-Net	1.27×	1.23×	-	<b>1.03</b> ×
ResNet50	1.20×	1.25×	-	<b>1.05</b> ×

**Within 6% of optimal cost** (geomean)

43x speedup for ResNet50  
440x speedup for MobileNet



# Checkmate

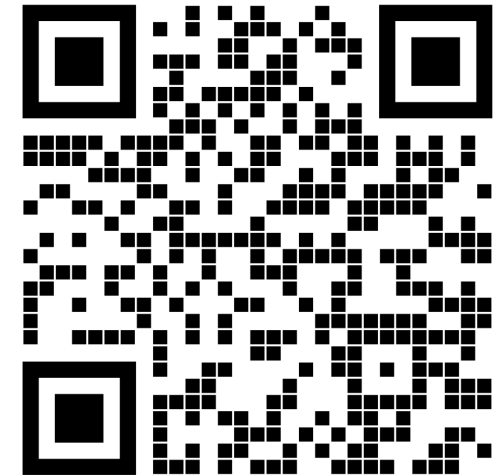


Code and paper:  
[checkmateai.github.io](https://github.com/checkmateai)

Email me:  
[parasj@berkeley.edu](mailto:parasj@berkeley.edu)

## Key ideas:

- GPU memory limits are preventing the development of new deep learning models.
- We present the first general solution for optimal & near-optimal graph rematerialization.
- Formulation supports **arbitrary DAGs** and is both **hardware-aware** and **memory-aware**
- Integration with just **one line of code**



```
train_iteration = checkmate.compile_tf2(  
    model, loss, optimizer, input_shape, label_shape)
```